

WORKBOOK

PACEMAKER®

Geometry



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P A C E M A K E R®

Geometry

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Pacemaker® Geometry, First Edition

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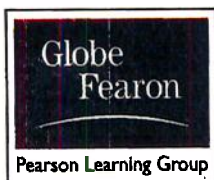
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A Note to the Student

The exercises in this workbook go along with your *Pacemaker*[®] *Geometry* textbook. This workbook will give you a chance to review concepts you learned in your textbook, to practice skills, and to think critically.

Set goals for yourself and try to meet them as you complete each activity. The more you practice, the more you will remember. Being able to remember and apply information is a skill that will help you succeed in school, at work, and in life.





Your critical thinking skills will be challenged. You will need to think beyond what you learned in your textbook. The critical thinking activities provide you with the opportunity to put the information you have learned to use.

Your textbook is a great source of information. By completing the activities in this workbook, you will learn a lot about geometry. The real value of learning geometry will be seen when you have mastered these skills and put your new knowledge to work!

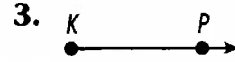
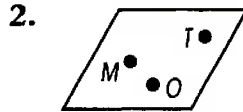
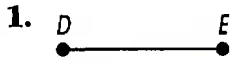
1 Geometric Figures and Distance Exercise 1

Lessons 1.1 and 1.4

This chart shows the basic geometric figures.

point D	$\bullet D$	plane ABC	
line segment AB		line AB	
ray BA			

Name each figure.



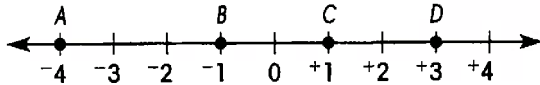
Draw each figure.

4. line AB

5. \overleftrightarrow{MS}

6. plane TRS

Use the number line below for exercises 7–12.



Find the distance between each set of points.

7. Points C and D

8. Points A and B

9. Points A and C

Find the length of each line segment.

10. \overline{AC}

11. \overline{BD}

12. \overline{AD}

CRITICAL THINKING

What does the symbol \overrightarrow{KB} mean? Which point is the endpoint?

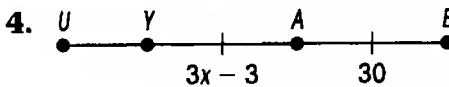
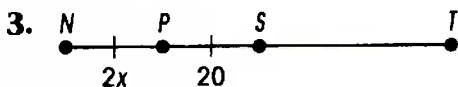
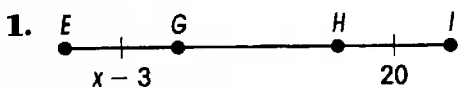
1 Congruent Line Segments Exercise 2

Lesson 1.5

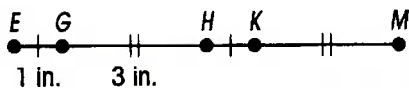
Line segments that have the same lengths are congruent line segments. In the diagram below, $\overline{AB} \cong \overline{CD}$. So, $AB = CD$.



Find the value of x in each diagram.



Use the diagram below to complete exercises 5–8.



5. $\overline{EG} \cong$ _____

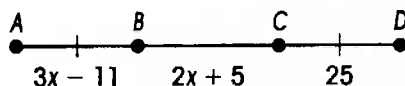
6. $\overline{KM} \cong$ _____

7. Find the length of \overline{HK} .

8. Find the length of \overline{KM} .

CRITICAL THINKING

Look at \overline{AD} on the right. Find the length of \overline{BC} .

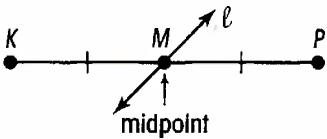


1 Midpoint of a Line Segment Exercise 3

Lesson 1.6

The midpoint of a line segment divides the line segment into two congruent parts.

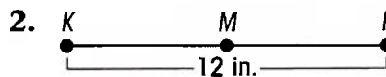
Because M is the midpoint of \overline{KP} , $\overline{KM} \cong \overline{MP}$ and line l is a bisector of \overline{KP} .



Point M is the midpoint of \overline{KP} . Find the length of \overline{MP} in each diagram.

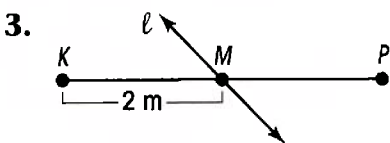


$MP = \underline{\hspace{2cm}}$

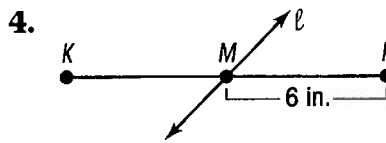


$MP = \underline{\hspace{2cm}}$

Line l bisects \overline{KP} at point M . Find the length of \overline{KP} in each diagram.



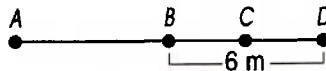
$KP = \underline{\hspace{2cm}}$



$KP = \underline{\hspace{2cm}}$

Use the diagram below to complete exercises 5–8.

Point C is the midpoint of \overline{BD} .
Point B is the midpoint of \overline{AD} .



5. $BC = \underline{\hspace{2cm}}$

6. $CD = \underline{\hspace{2cm}}$

7. $AB = \underline{\hspace{2cm}}$

8. $AC = \underline{\hspace{2cm}}$

CRITICAL THINKING

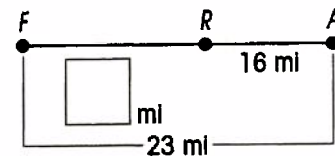
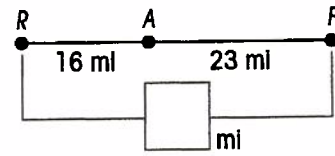
A line bisects \overline{GH} at point K . The length of \overline{KH} is 11 m. Find the length of \overline{GH} . (Hint: Draw a diagram first.)

1 Problem-Solving Strategy: Draw a Diagram Exercise 4

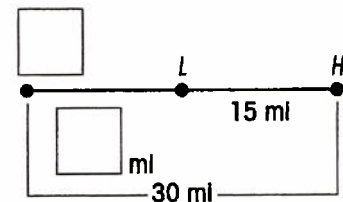
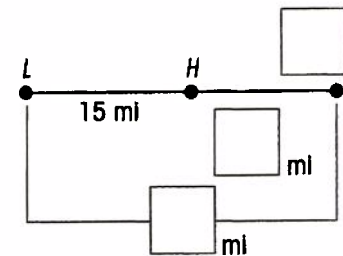
Lesson 1.8

To solve each problem, first complete the diagrams. Then, solve the problem. There are two possible answers.

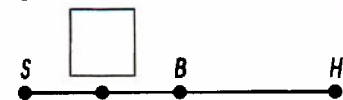
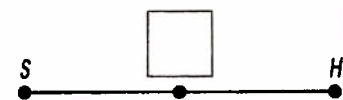
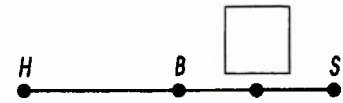
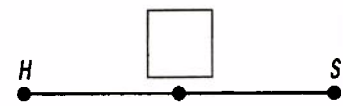
1. Along a straight highway are exit points R , F , and A . The exits are not in that order. The distance from point R to point A is 16 miles. The distance from point F to point A is 23 miles. Find the distance from point F to point R .



2. A bus makes stops at points S , L , and H along a straight road. The points are not in that order. The distance from point L to point H is 15 miles. The distance from point S to point H is 30 miles. Find the distance from point S to point L .



3. Along a straight road is a store (S), a bank (B), a gas station (G), and a house (H). The buildings are not in that order. The bank is between the house and the store. The gas station is between the store and the bank. What could the third building be?



2 Classifying Angles

Exercise 6

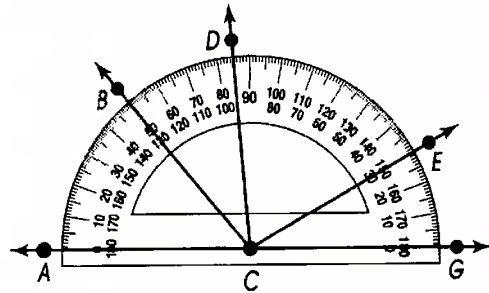
Lessons 2.1 and 2.2

There are four types of angles.

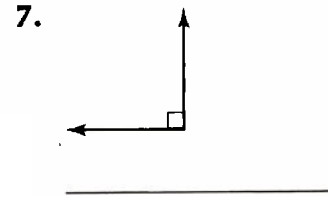
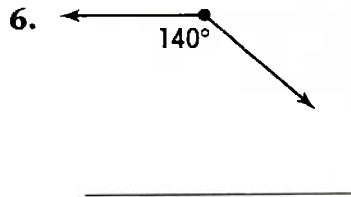
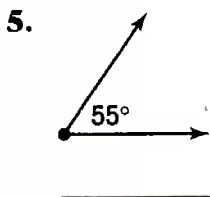
- acute angle greater than 0° and less than 90°
- right angle equals 90°
- obtuse angle greater than 90° and less than 180°
- straight angle equals 180°

Look at the protractor on the right.
Find the measure of each angle.

1. $\angle ACD$ _____ 2. $\angle ACE$ _____
3. $\angle GCB$ _____ 4. $\angle GCE$ _____



Classify each angle. Write *acute*, *right*, *obtuse*, or *straight*.



Draw each angle. Then, name the angle.

8. obtuse

9. acute

10. straight

CRITICAL THINKING

The measure of an obtuse angle is three times the measure of an acute angle. What could be the measure of the acute angle? (Hint: There is more than one possible answer.)

2 Adding and Subtracting Angle Measures

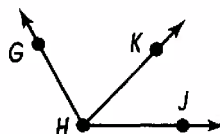
Exercise 7

Lesson 2.4

Adjacent angles are two angles with a common vertex and a common ray.

$\angle GHK$ and $\angle KHJ$ are adjacent angles.

$$m\angle GHK + m\angle KHJ = m\angle GHJ$$

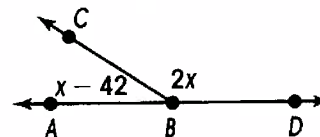


Draw a diagram. Find the measure of each angle.

- $\angle ABC$ is adjacent to $\angle CBD$.
 $\angle ABC$ is 90° and $\angle CBD$ is 45° .
 Find the measure of $\angle ABD$.
- $\angle MPQ$ is adjacent to $\angle QPS$.
 $\angle MPQ$ is 23° and $\angle QPS$ is 48° .
 Find the measure of $\angle MPS$.
- $\angle LMK$ is adjacent to $\angle KMP$.
 $\angle LMP$ is 139° and $\angle KMP$ is 67° .
 Find the measure of $\angle LMK$.

CRITICAL THINKING

Look at the diagram on the right. $\angle ABD$ is a straight angle. Write and solve an equation to find the measure of $\angle ABC$ and $\angle CBD$.



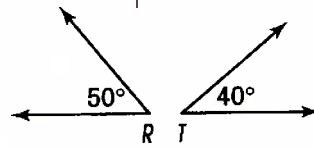
2 Complementary and Supplementary Angles Exercise 8

Lesson 2.5

A pair of angles whose measures have a sum of 90° are complementary angles.

A pair of angles whose measures have a sum of 180° are supplementary angles.

1. Decide if $\angle R$ and $\angle T$ are complementary or supplementary angles. Explain your answer.



Find the angle that is complementary to each angle.

- | | | |
|---------------|-------------------|---------------|
| 2. 10° | 3. 55° | 4. 73° |
| 5. 28° | 6. γ° | 7. 39° |

Find the angle that is supplementary to each angle.

- | | | |
|-----------------|----------------|-----------------|
| 8. 80° | 9. 120° | 10. 135° |
| 11. 105° | 12. 66° | 13. 171° |

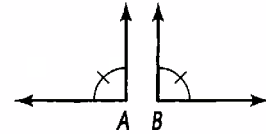
CRITICAL THINKING

An angle is 48° more than its supplementary angle. Find the measure of each angle. (Hint: Write an equation first. Let x represent the measure of the supplementary angle.)

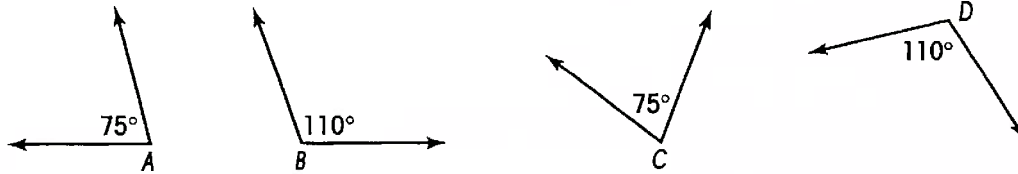
2 Congruent Angles Exercise 9

Lesson 2.6

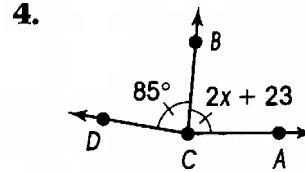
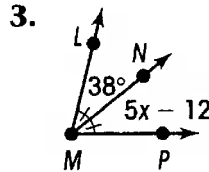
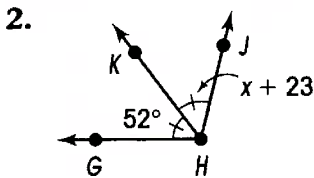
Congruent angles are angles that have the same measure.
 Because $\angle A \cong \angle B$, then $m\angle A = m\angle B$. A small mark on the diagram tells you that the angles are congruent.



1. Which of the following angles are congruent?



Find the value of x in each diagram.



Use the diagram on the right for exercises 5–8.

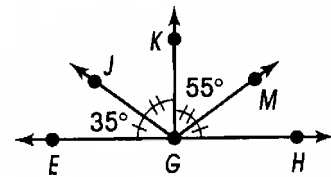
Find the measure of each angle.

5. $\angle JGK$ _____

6. $\angle MGH$ _____

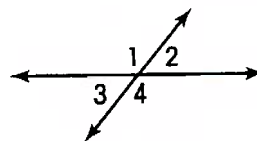
7. $\angle EGK$ _____

8. $\angle HGK$ _____



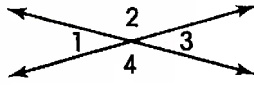
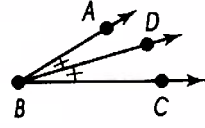
CRITICAL THINKING

Use the diagram to find the measure of $\angle 4$.
 The measure of $\angle 3$ is 50° . $\angle 3$ and $\angle 4$ are supplementary.



2 Vertical Angles and Angle Bisectors Exercise 10

Lessons 2.7 and 2.8

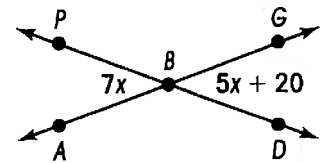
<p>Vertical Angles</p>  <p style="margin-left: 40px;"> $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$ </p>	<p>Angle Bisector</p> <p>\overrightarrow{BD} bisects $\angle ABC$.</p>  <p style="margin-left: 40px;">$\angle ABD \cong \angle DBC$</p>
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Use the diagram on the right to find the measure of each angle in exercises 1–3.

1. $\angle PBA$

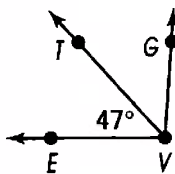
2. $\angle ABD$

3. $\angle GBP$



Find the measure of each angle.

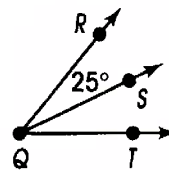
4.



\overrightarrow{VT} bisects $\angle EVG$.

Find the measure of $\angle TVG$.

5.



\overrightarrow{QS} bisects $\angle RQT$.

Find the measure $\angle SQT$.

CRITICAL THINKING

\overrightarrow{AB} intersects \overrightarrow{CD} at point E . If $\angle AEC$ is 78° , what is the measure of $\angle AED$?
 (Hint: Draw a diagram.)

2 Problem-Solving Strategy: Make a Table **Exercise 11**

Lesson 2.10

Make a table to solve each problem. The first table is started for you.

1. After the cast is off your leg, your physical therapist wants you to bend your knee 12° more each week. You need to bend your knee from 180° to 120° . How many weeks will it take you to get to 120° ?

<i>Weeks</i>						
<i>Angle of elbow</i>						

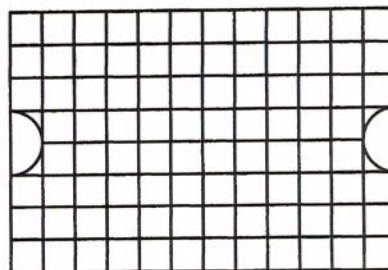
2. After Phyllis has her cast off her leg, her physical therapist wants her to bend her knee 4° more each day. She needs to bend her knee from 180° to 152° . How many days will it take Phyllis to get to 152° ?
3. After his elbow surgery, Mark is put into a cast that holds his arm at a 90° angle. Once his cast is off, he will do exercises to straighten his arm. His doctor wants him to straighten his elbow by about 8° each week. How many weeks will it take Mark to go from 90° to 162° ?

2 Problem-Solving Application: Angles and Sports **Exercise 12**

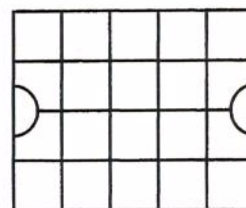
Lesson 2.11

Solve each problem. Show your work. If the puck gets stuck in a corner, tell which corner.

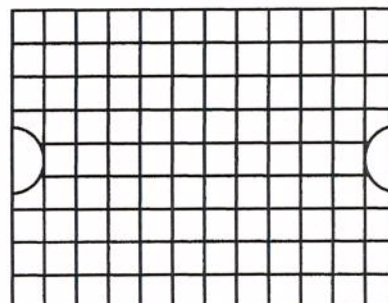
- Kim plays air hockey on an 8-ft by 12-ft table. She hits the puck from the lower left corner at a 45° angle. How many times will the puck hit the side of the table before it lands in a pocket?



- Tova plays air hockey on an 4-ft by 5-ft table. She hits the puck from the lower left corner at a 45° angle. How many times will the puck hit the side of the table before it lands in a pocket?



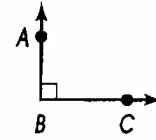
- Jon plays air hockey on a 9-ft by 12-ft table. He hits the puck from the lower left corner at a 45° angle. Will the puck land in a pocket?



3 Reasoning Exercise 13

Lessons 3.1, 3.2, and 3.3

- You can use inductive reasoning to continue a pattern.
- The marks on a diagram help you to reach a conclusion.
 $\angle ABC$ is a right angle.
- The *then* part of a statement is the conclusion.
 If $\angle ABC$ is a right angle, *then* $m\angle ABC$ is 90.

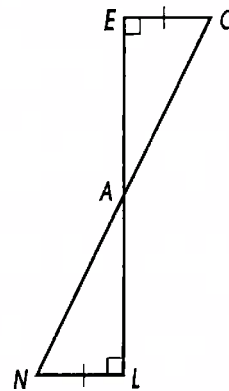


Continue each pattern.

1. 30, 28, 33, 31, 36, _____, _____

2. • • • _____

Use the diagram on the right for exercises 3–5.



3. Write a conclusion about $\angle CAE$ and $\angle NAL$.

4. Write a conclusion about CE and NL .

5. Write a conclusion about $\angle CEA$ and $\angle NLA$.

Write a conclusion for each conditional statement.

6. If two angles are congruent, then _____.

7. If \overrightarrow{MR} bisects $\angle LMN$, then _____.

CRITICAL THINKING

Give two possible conclusions for the following conditional statement.

If two adjacent angles combine to form a straight angle, then _____.

3 Properties of Equality and Congruence

Exercise 14

Lessons 3.4 and 3.5

Properties of Equality		Properties of Congruence	
Addition	If $x = y$, then $x + 2 = y + 2$.	Reflexive	$\overline{CD} \cong \overline{CD}$ and $\angle A \cong \angle A$
Subtraction	If $x = y$, then $x - 2 = y - 2$.	Symmetric	If $\overline{CD} \cong \overline{EF}$, then $\overline{EF} \cong \overline{CD}$. If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Multiplication	If $x = y$, then $x \cdot 2 = y \cdot 2$.	Transitive	If $\overline{CD} \cong \overline{EF}$ and $\overline{EF} \cong \overline{GH}$, then $\overline{CD} \cong \overline{GH}$. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.
Division	If $x = y$, then $x \div 2 = y \div 2$		
Substitution	If $x = 3$, then you can use 3 for x . So, $x + y = 3 + y$.		

Name the property of equality you can use to reach each conclusion.

- If $AT = MG$, then $AT + 25 = MG + 25$. _____
- If $AB = CD$, then $AB \div 5 = CD \div 5$. _____
- If $XY = ST$, then $XY - 4 = ST - 4$. _____
- If $AB + BC = 40$ and $AB = CD$, then $CD + BC = 40$. _____

Name the property of congruence you can use to reach each conclusion.

- $\overline{BD} \cong \overline{BD}$ _____
- If $\angle D \cong \angle P$ and $\angle P \cong \angle K$, then $\angle D \cong \angle K$. _____

Use a property of congruence or a property of equality to write the conclusion. Name the property you used.

- If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then _____.

Property: _____

CRITICAL THINKING

When you look in a mirror, you can see your reflection. Which property of congruence is that like?

3 Paragraph Proof **Exercise 15**

Lesson 3.6

You can write a proof in paragraph form. Begin with the information you are given. Then, tell how you can reach the statement you wish to prove.

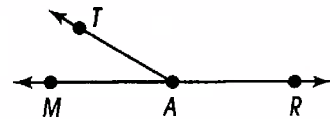
Complete the paragraph proof. Use the terms and symbols below. Each term or symbol is used once.

supplementary, straight, 180, $m\angle MAT$, $m\angle TAR$

You are given:

$\angle MAR$ is a straight angle.

Prove: $\angle MAT$ and $\angle TAR$ are supplementary angles.



You are given that $\angle MAR$ is a _____ angle.

This means that $m\angle MAR =$ _____ . You know that

$m\angle MAR = m\angle MAT +$ _____ . You can substitute 180 for

$m\angle MAR$ in this equation. $180 =$ _____ + $m\angle TAR$ This means

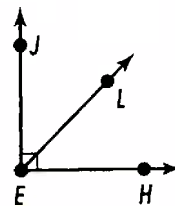
that $\angle MAT$ and $\angle TAR$ are _____ angles.

CRITICAL THINKING

Write a paragraph proof for the following.

You are given: $\angle JEH$ is a right angle.

Prove: $\angle JEL$ and $\angle LEH$ are complementary angles.



3 Two-Column Proof

Exercise 16

Lesson 3.7

You can write a proof in two-column form.

The first column is for statements.
 The second column is for reasons. The reasons tell you why the statement is true. Use what is given, definitions, properties, postulates, and theorems for the reasons.

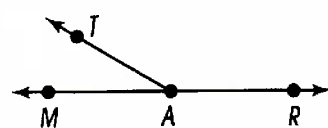
Complete the two-column proof. Use the terms and symbols below. Each term or symbol is used once.

Angle, Definition, 180, Given, straight angle

You are given:

$\angle MAR$ is a straight angle.

Prove: $\angle MAT$ and $\angle TAR$ are supplementary angles.



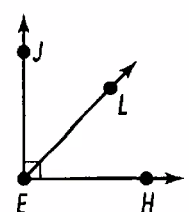
Statements	Reasons
1. $\angle MAR$ is a straight angle.	1. _____
2. $m\angle MAR = 180$	2. Definition of a _____
3. $m\angle MAT + m\angle TAR = m\angle MAR$	3. _____ Addition Postulate
4. $m\angle MAT + m\angle TAR =$ _____	4. Substitution Property of Equality
5. $\angle MAT$ and $\angle TAR$ are supplementary angles.	5. _____ of supplementary angles

CRITICAL THINKING

Write a two-column proof for the following.

You are given: $\angle JEH$ is a right angle.

Prove: $\angle JEL$ and $\angle LEH$ are complementary angles.



3 Problem-Solving Skill: Indirect Proof - Exercise 17

Lesson 3.9

To write an indirect proof:

- Assume that the **opposite** of what you want to prove is true.
- Prove that the assumption is false.
- Then, what you want to prove must be true.

Write the opposite of each statement.

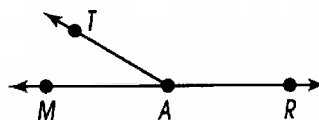
1. $\angle 1$ and $\angle 2$ are vertical angles. _____
2. \vec{JK} bisects $\angle MJN$. _____
3. $\overline{AB} \cong \overline{CD}$ _____
4. Complete the indirect proof.

Use these terms and symbols to complete the indirect proof.
Each term or symbol is used once.

straight, \neq , false, supplementary, are not

You are given:

$\angle MAR$ is a straight angle.



Prove: $\angle MAT$ and $\angle TAR$ are supplementary angles.

Assume: $\angle MAT$ and $\angle TAR$ _____ supplementary angles.

If $\angle MAT$ and $\angle TAR$ are not supplementary angles, then

$m\angle MAT + m\angle TAR$ _____ 180.

This contradicts the given fact that $\angle MAR$ is a _____ angle.

Thus, what you assume is _____.

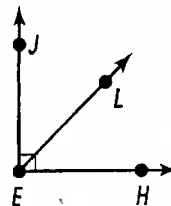
So, you prove that $\angle MAT$ and $\angle TAR$ are _____ angles.

CRITICAL THINKING

Write an indirect proof for the following.

You are given: $\angle JEH$ is a right angle.

Prove: $\angle JEL$ and $\angle LEH$ are complementary angles.



3 Problem-Solving Application: Flow Proof Exercise 18

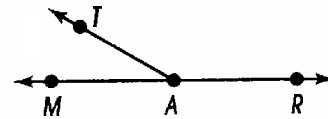
Lesson 3.10

You can write a flow proof. Place each statement and reason in a box. Use arrows to show how each statement leads to another.

Complete the flow proof. Use the terms and symbols below. Each term or symbol is used once.

Angle Addition Postulate, 180, straight angle, Definition

You are given: $\angle MAR$ is a straight angle.
 Prove: $\angle MAT$ and $\angle TAR$ are supplementary angles.

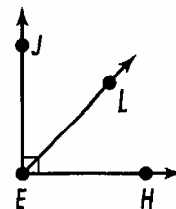


$\angle MAR$ is a straight angle. Given	
↓	
$m\angle MAR = 180$ Definition of a _____ _____	
↓	
$m\angle MAR = m\angle MAT + m\angle TAR$ _____	_____ = $m\angle MAT + m\angle TAR$ Substitution Property of Equality
	↓
	$\angle MAT$ and $\angle TAR$ are supplementary angles. _____ of supplementary angles

CRITICAL THINKING

Write a flow proof for the following.

You are given: $\angle JEH$ is a right angle.
 Prove: $\angle JEL$ and $\angle LEH$ are complementary angles.



4 Perpendicular Lines and the Perpendicular Bisector

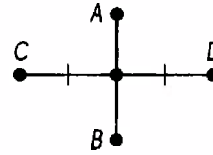
Exercise 19

Lessons 4.1 and 4.2

Lines, line segments, or rays that intersect and form a right angle are perpendicular.

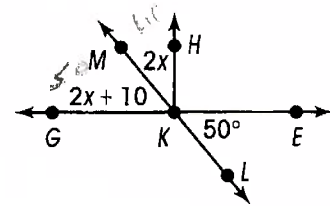
\overline{AB} is the perpendicular bisector of \overline{CD} .

\overline{AB} bisects \overline{CD} and \overline{AB} is perpendicular to \overline{CD} .



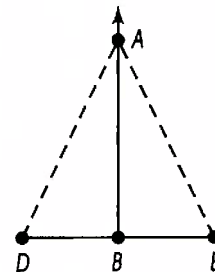
In the diagram, $\overline{HK} \perp \overline{GE}$. Find the measure of each angle.

1. $\angle GKH$ _____
2. $\angle MKG$ _____
3. $\angle MKH$ _____
4. $\angle GKL$ _____



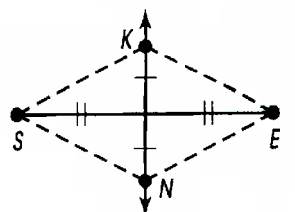
In the diagram, \overline{AB} is the perpendicular bisector of \overline{DE} .

5. Find the value of x if $BD = 3x + 10$ and $BE = x + 40$.
6. Find the value of x if $AD = 40$ and $AE = x + 8$.
7. Find the distance between point A and point E if $AD = 2x - 12$ and $AE = x + 16$.
8. Find the distance between point A and point D if $AD = 3x - 18$ and $AE = x + 12$.



CRITICAL THINKING

In the diagram on the right, $\overleftrightarrow{KN} \perp \overleftrightarrow{SE}$.
 $SN = 3x - 27$ and $KE = x + 43$.
 Find SK .

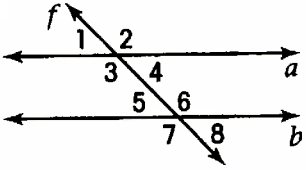


4 Parallel Lines With Transversals Exercise 20

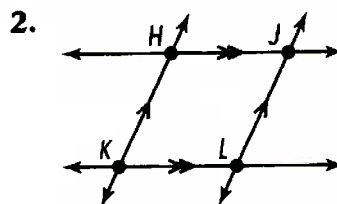
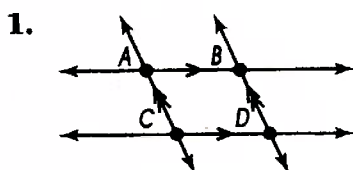
Lessons 4.3 and 4.4

In the diagram on the right, $a \parallel b$.

$\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles.
 $\angle 3$ and $\angle 5$ are same-side interior angles.
 $\angle 3$ and $\angle 6$ are alternate interior angles.

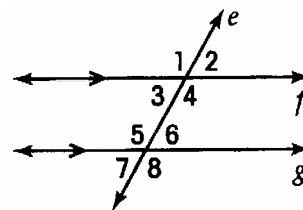


Identify the parallel lines in each diagram.



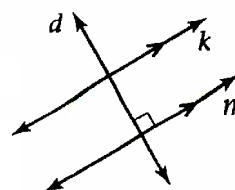
Use the diagram on the right. Name all angles for each exercise.

3. interior angles
4. same-side interior angles
5. alternate interior angles
6. vertical angles



CRITICAL THINKING

Decide which lines in the diagram on the right are perpendicular. Then, decide which lines are parallel.



4 Measures of Interior Angles

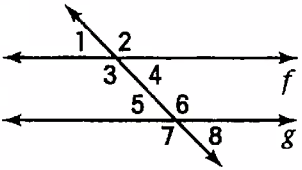
Exercise 21

Lessons 4.5 and 4.6

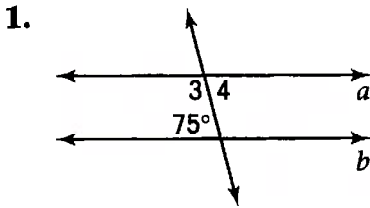
In the diagram, $f \parallel g$.

Alternate interior angles are congruent.
So, $\angle 3 \cong \angle 6$.

Same-side interior angles are supplementary.
So, $m\angle 3 + m\angle 5 = 180$.

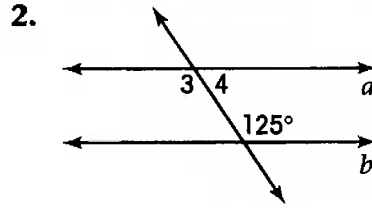


In each diagram, $a \parallel b$. Find the measure of $\angle 3$ and $\angle 4$.



$\angle 3$ is _____.

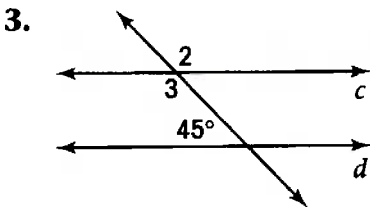
$\angle 4$ is _____.



$\angle 3$ is _____.

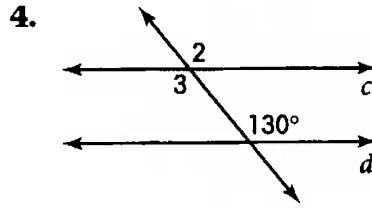
$\angle 4$ is _____.

In each diagram, $c \parallel d$. Find the measure of $\angle 2$ and $\angle 3$.



$\angle 2$ is _____.

$\angle 3$ is _____.

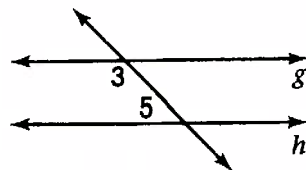


$\angle 2$ is _____.

$\angle 3$ is _____.

CRITICAL THINKING

In the diagram on the right, $g \parallel h$. Find the measure of $\angle 5$ and $\angle 3$ if $\angle 5 = x$ and $\angle 3 = 3x$.



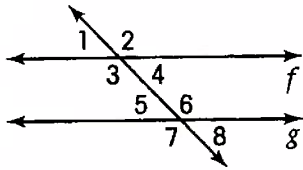
4 Corresponding Angles Exercise 22

Lesson 4.7

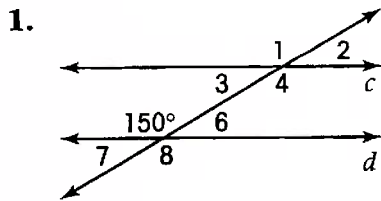
In the diagram, $f \parallel g$.

$\angle 1$ is an exterior angle.
 $\angle 5$ is an interior angle.
 $\angle 1$ and $\angle 5$ are corresponding angles.
 Corresponding angles are congruent.

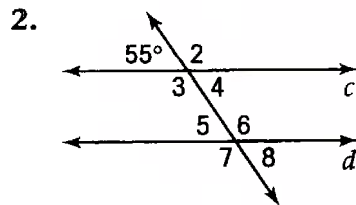
So, $\angle 1 \cong \angle 5$.



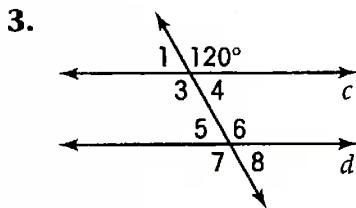
In each diagram, $c \parallel d$. Find the measure of each angle named.



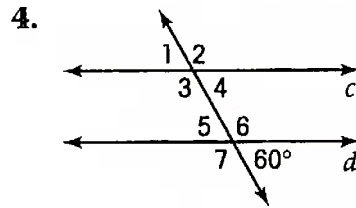
$\angle 1$ is _____.



$\angle 5$ is _____.



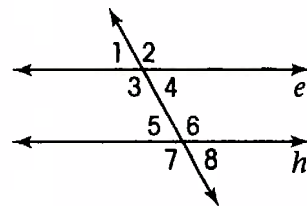
$\angle 7$ is _____.



$\angle 3$ is _____.

CRITICAL THINKING

In the diagram on the right, $e \parallel h$. If $\angle 3$ is $2x - 25$ and $\angle 7$ is $x + 36$, find the measure of $\angle 8$.



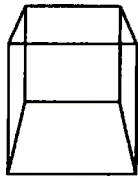
4 Problem-Solving Skill:
Draw a One-Point Perspective

Exercise 23

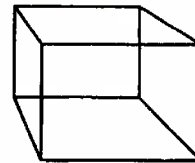
Lesson 4.9

Locate and draw the vanishing point. Show your work.

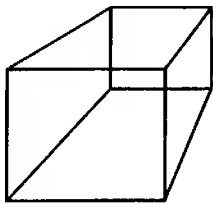
1.



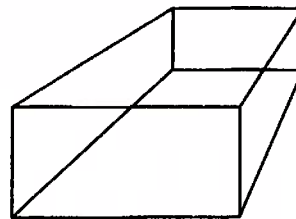
2.



3.

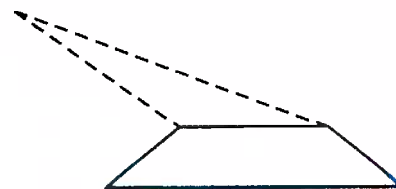


4.



CRITICAL THINKING

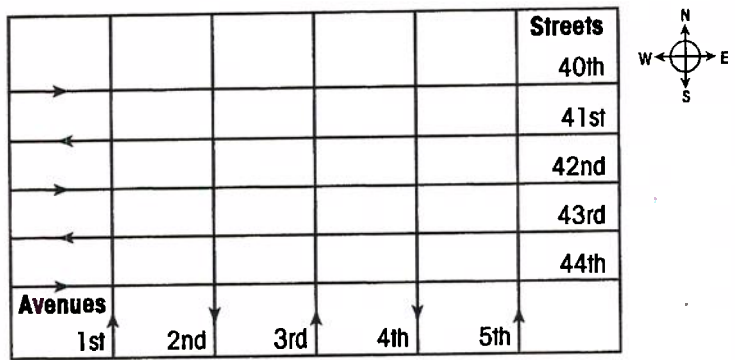
Devon began to draw a one-point perspective of a three-dimensional object. Complete his drawing.



4 Problem-Solving Application: Taxicab Routes Exercise 24

Lesson 4.10

Use the map below. Solve each problem.



1. Meg gets into a taxicab at 41st Street and 4th Avenue. She wants to go to 43rd Street and 1st Avenue. Find the most direct route.

2. Bruce gets into a taxicab at 44th Street and 1st Avenue. He wants to go to 40th Street and 5th Avenue. How many different routes can the taxicab take?

3. Juanita wants to drive from 44th Street and 5th Avenue to 42nd Street and 2nd Avenue. Find the most direct route.

5 Angles of a Triangle Exercise 25

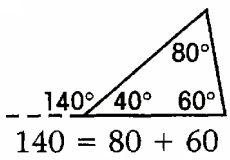
Lessons 5.1, 5.2, and 5.3

You can classify triangles by their angles.

- Acute Triangle All angles are less than 90° .
- Obtuse Triangle One angle is greater than 90° .
- Equiangular Triangle All angles are equal.

Remember these theorems about triangles.

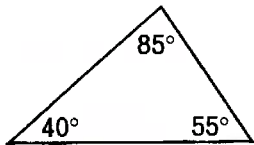
- The sum of the angles in a triangle is 180° .
- The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.



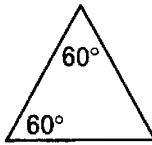
$140 = 80 + 60$

Classify each triangle. Write *acute*, *obtuse*, or *equiangular*.

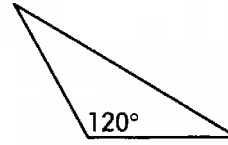
1.



2.

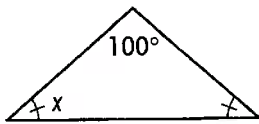


3.

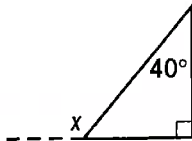


Find the value of x .

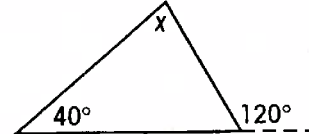
4.



5.



6.



CRITICAL THINKING

The exterior angle of a triangle has a measure of 67° . What are the measures of the two remote interior angles? There is more than one possible answer. (Hint: Draw a diagram.)

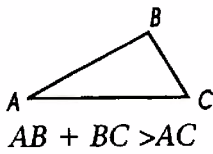
5 Sides of a Triangle Exercise 26

Lessons 5.4 and 5.5

You can classify triangles by their sides.

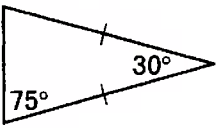
- Scalene Triangle No sides are congruent.
- Isosceles Triangle Two sides are congruent.
- Equilateral Triangle All sides are congruent.

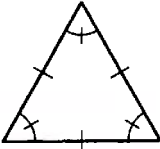
Remember that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

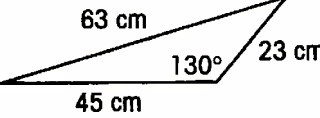


$AB + BC > AC$

Classify each triangle by its angles and sides.

1. 

2. 

3. 

Can you form a triangle with the given segment lengths? If so, classify each triangle. Write *scalene*, *isosceles*, or *equilateral*.

4. 14 yd, 14 yd, 14 yd

5. 14 m, 15 m, 36 m

6. 5 in., 5 in., 6 in.

7. 7 cm, 8 cm, 9 cm

CRITICAL THINKING

Can segments with lengths 4 cm, 5 cm, and 9 cm form a triangle? How do you know?

5 Side-Angle Relationship

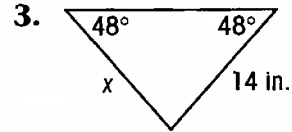
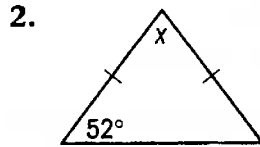
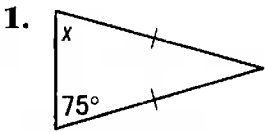
Exercise 27

Lessons 5.6 and 5.7

Remember these theorems about triangles.

- **Isosceles Triangle Theorem**
If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.
- **Opposite Side-Angle Theorem**
The longest side of a triangle is opposite the largest angle.

Find the value of x .

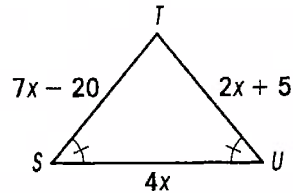


Find the length of each side of $\triangle STU$.

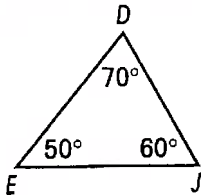
4. \overline{ST} _____

5. \overline{TU} _____

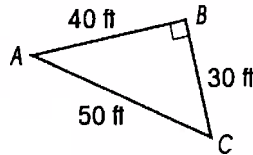
6. \overline{SU} _____



7. Name the longest side.

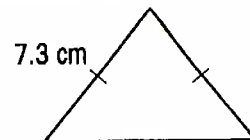


8. Name the largest angle.



CRITICAL THINKING

What is the largest possible integer length for the base of this triangle?



5 Congruent Triangles

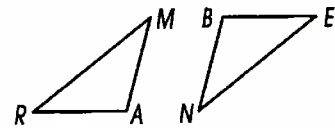
Exercise 28

Lessons 5.8, 5.9 and 5.10

You can show that two triangles are congruent.

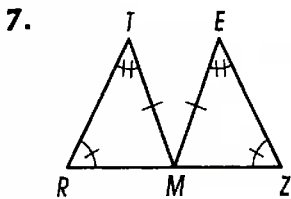
- **Angle-Side-Angle Postulate (ASA)**
Two triangles are congruent if two corresponding angles and the included side are congruent.
- **Side-Angle-Side Postulate (SAS)**
Two triangles are congruent if two corresponding sides and the included angle are congruent.
- **Angle-Angle-Side Theorem (AAS)**
Two triangles are congruent if two corresponding angles and a corresponding side are congruent.

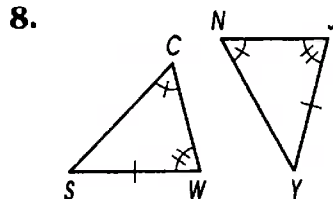
In the diagram, $\triangle MAR \cong \triangle NBE$. Complete each statement.

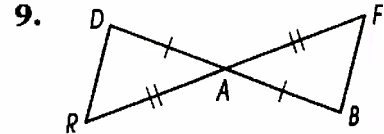


- | | |
|--------------------------------|--------------------------------|
| 1. $\angle A \cong$ _____ | 2. $\angle R \cong$ _____ |
| 3. $\angle M \cong$ _____ | 4. $\overline{MR} \cong$ _____ |
| 5. $\overline{MA} \cong$ _____ | 6. $\overline{RA} \cong$ _____ |

Write a congruence statement for each pair of triangles. Name the postulate or theorem that you used. Write AAS, SAS, or ASA.







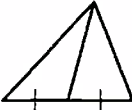
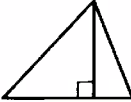
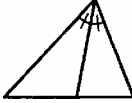
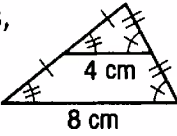
CRITICAL THINKING

Draw a triangle. Label it $\triangle ABC$. Choose one of the postulates above. Use the postulate to construct $\triangle DEF$ congruent to $\triangle ABC$. Construct only those congruent parts named in the postulate.

5 Medians, Altitudes, and Bisectors Exercise 29

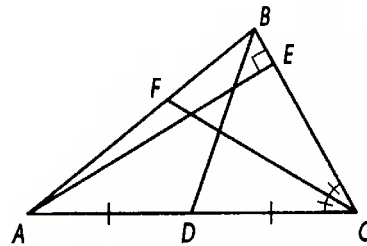
Lessons 5.11 and 5.12

Remember the special line segments in a triangle.

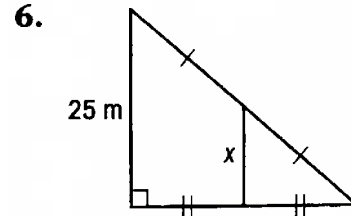
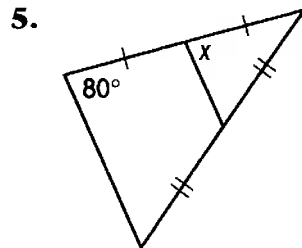
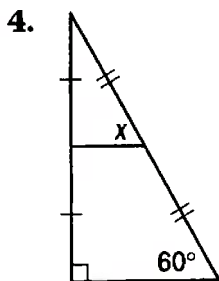
- median Joins a vertex to the midpoint of the opposite side 
- altitude Joins a vertex to the line containing the opposite side and is perpendicular to that side 
- angle bisector Joins a vertex to the opposite side and bisects the angle 
- midsegment Joins the midpoints of any two sides, is parallel to the third side, and is half its length 

Use $\triangle ABC$ to name a line segment for each term.

1. median _____
2. altitude _____
3. angle bisector _____



Find the value of x in each triangle.



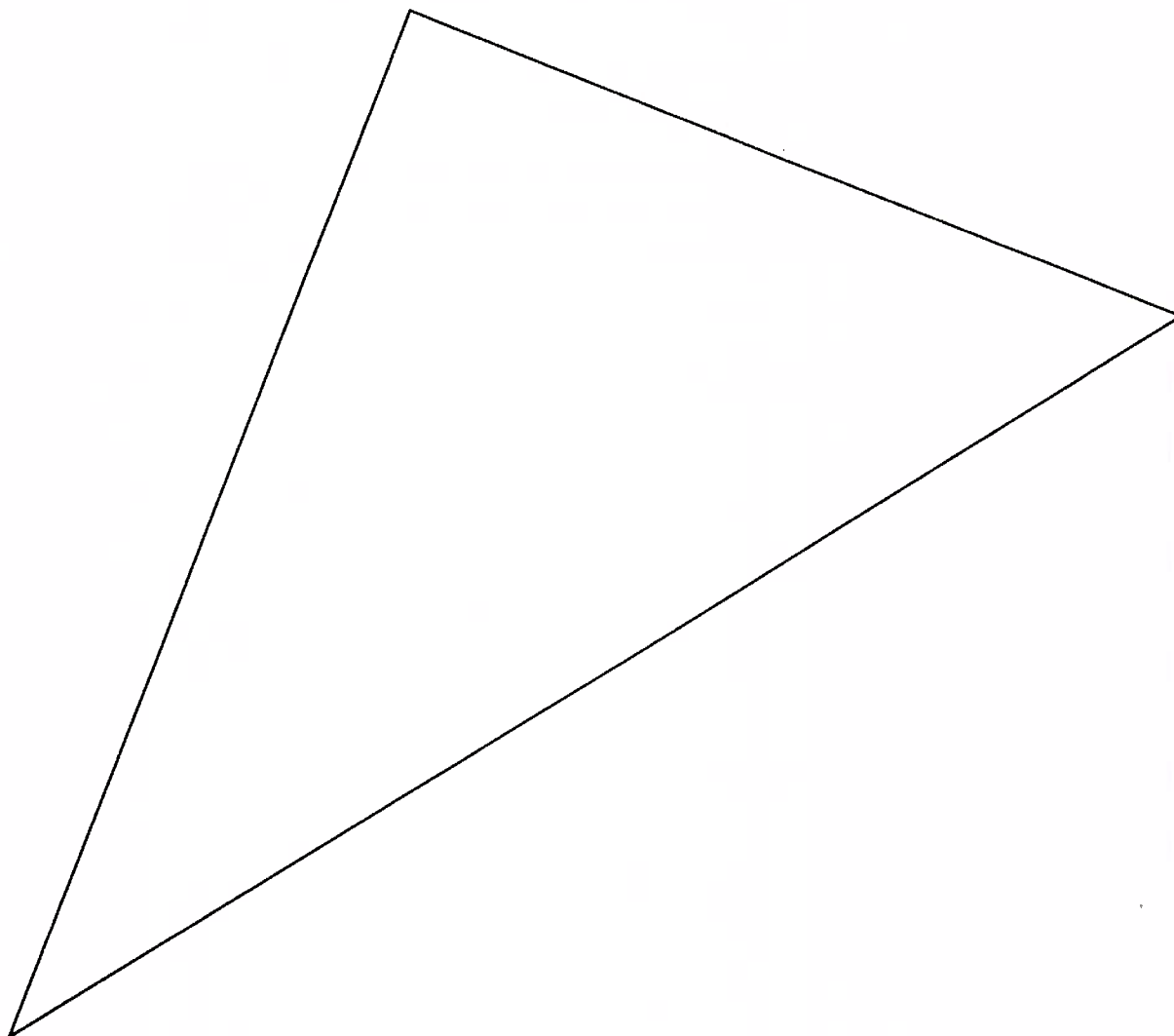
CRITICAL THINKING

Draw a right triangle. Find and label the three altitudes.

5 Problem-Solving Skill: Find the Centroid Exercise 30

Lesson 5.14

Make a model of this triangle on heavy paper or cardboard.
Find the centroid. Then, balance the triangle on a pencil
point at the centroid.

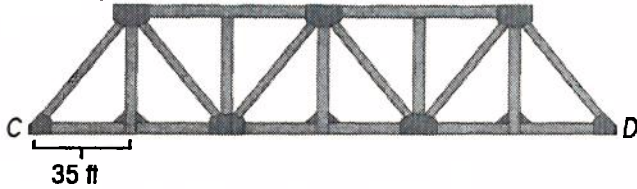


5 Problem-Solving Application: Engineering Exercise 31

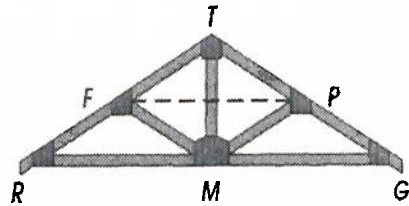
Lesson 5.15

Solve each problem. Show your work.

1. This truss is part of a bridge. It is made up of ten congruent isosceles right triangles. How long is the distance from C to D ?



2. This truss supports a roof. The distance from point F to point P is 26 feet. Point F and point P are midpoints of \overline{RT} and \overline{TG} . What is the span, \overline{RG} , of this roof?



3. The rafters of a roof must meet at a 110° angle. The rafters form part of an isosceles triangle. What is the measure of one of the base angles? (Hint: Draw a diagram.)

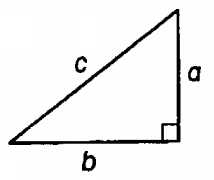
6 Pythagorean Theorem

Exercise 32

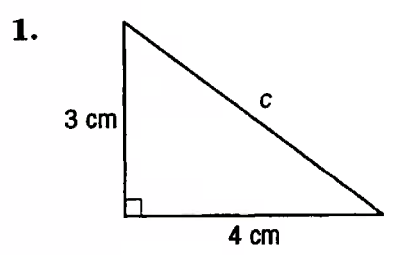
Lessons 6.4 and 6.5

Remember the Pythagorean Theorem.

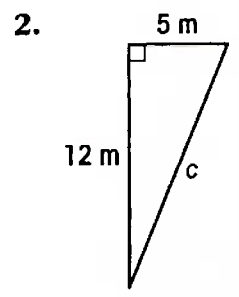
$$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

$$a^2 + b^2 = c^2$$


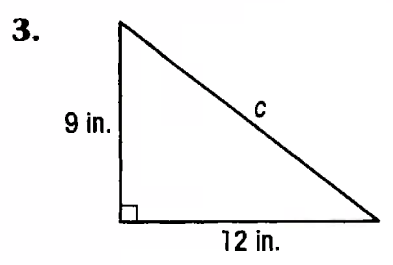
Find the unknown length of the side of each triangle.



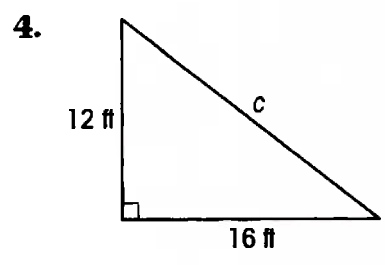
$c =$ _____



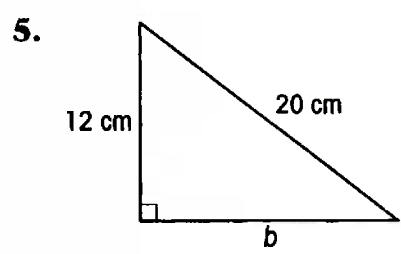
$c =$ _____



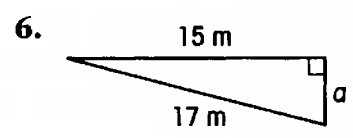
$c =$ _____



$c =$ _____



$b =$ _____



$a =$ _____

CRITICAL THINKING

Is a triangle with sides 6 cm, 8 cm, and 10 cm a right triangle?
How do you know?

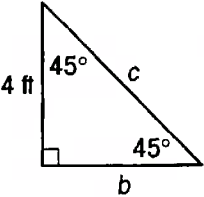
6 Special Right Triangle: 45°-45°-90° Exercise 33

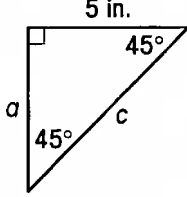
Lesson 6.6

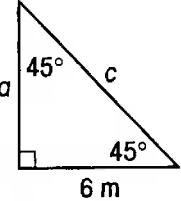
If you know the length of one side of a 45°-45°-90° right triangle, you can find the lengths of the other two sides.

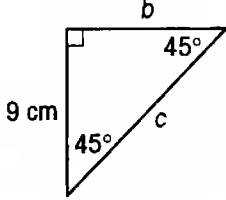
leg = leg
 hypotenuse = leg • $\sqrt{2}$

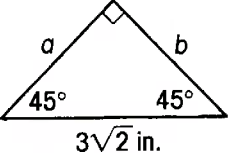
Find the unknown lengths of the sides of each triangle. The first one is done for you.

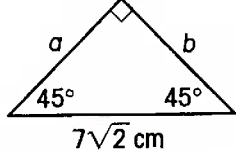
1.  $b = \underline{4 \text{ ft}}$
 $c = \underline{4 \cdot \sqrt{2} = 4\sqrt{2} \text{ ft}}$

2.  $a = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

3.  $a = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

4.  $b = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

5.  $a = \underline{\hspace{2cm}}$
 $b = \underline{\hspace{2cm}}$

6.  $a = \underline{\hspace{2cm}}$
 $b = \underline{\hspace{2cm}}$

CRITICAL THINKING

The leg of a 45°-45°-90° right triangle is $3\sqrt{2}$ in. long. Find the length of the hypotenuse.

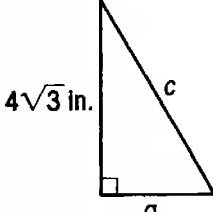
6 Special Right Triangle: 30°-60°-90° Exercise 34

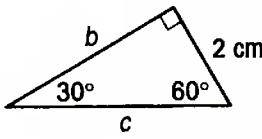
Lesson 6.7

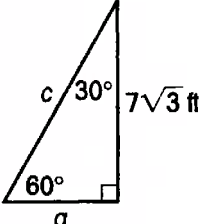
If you know the length of one side of a 30°-60°-90° triangle, you can find the lengths of the other two sides.

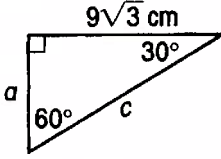
long leg = short leg • $\sqrt{3}$
 hypotenuse = 2 • short leg

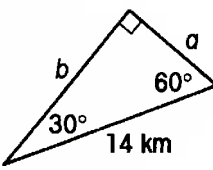
Find the unknown lengths of the sides of each triangle.
 The first one is done for you.

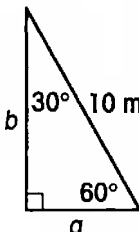
1.  $a = 4 \text{ in.}$
 $c = 2 \cdot 4 = 8 \text{ in.}$

2.  $b = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

3.  $a = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

4.  $a = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$

5.  $a = \underline{\hspace{2cm}}$
 $b = \underline{\hspace{2cm}}$

6.  $a = \underline{\hspace{2cm}}$
 $b = \underline{\hspace{2cm}}$

CRITICAL THINKING

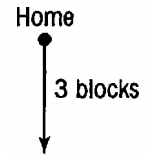
In a right triangle, the short leg is $3\sqrt{2}$ ft long. Find the length of the long side and the length of the hypotenuse.

**6 Problem-Solving Strategy:
Draw a Diagram****Exercise 35***Lesson 6.9*

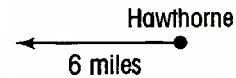
Draw a diagram to solve each problem.

The starting point is drawn for you.

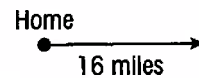
1. Tanya walks to school every day. From her home, she walks 3 blocks south. Then, she walks 4 blocks west to school. What is the shortest distance Tanya could travel if she could go directly from home to school?



2. Paterson Falls is 6 miles west of Hawthorne. Eight miles north of Paterson Falls is Lake Lincoln. What is the shortest distance from Lake Lincoln to Hawthorne?



3. Sam went for a long drive. From home, he drove 16 miles east. Then, he drove north. If Sam could drive the shortest distance home, it would be 20 miles. How far north did Sam drive?



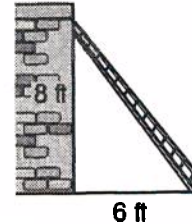
6 Problem-Solving Application: Indirect Measurement

Exercise 36

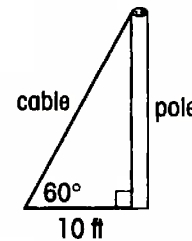
Lesson 6.10

Solve each problem. Show your work.

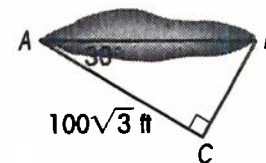
1. A mason is building a wall. His ladder is set up 8 feet against the wall. The bottom of the ladder is 6 feet from the wall. How long is the ladder? (Hint: $a^2 + b^2 = c^2$)



2. A cable is connected to a pole. It forms a 60° angle with the ground. At the bottom, the cable is 10 feet from the pole. What is the height of the pole? (Hint: long leg = short leg $\cdot \sqrt{3}$)

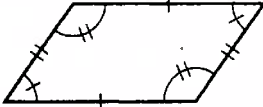


3. Yoshi swam across a lake but did not know how far he swam. He found that it was $100\sqrt{3}$ feet from point A to point C. Angle C was a right angle and angle A was 30° . Find the distance he swam across the lake. (Hint: hypotenuse = $2 \cdot$ short leg)

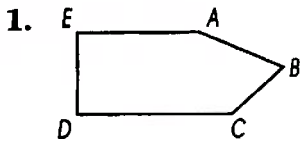


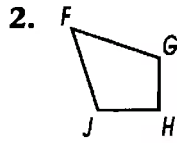
7 Polygons and Parallelograms Exercise 37

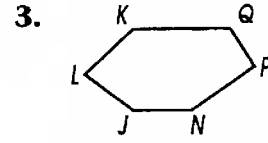
Lessons 7.1 and 7.2

<p>This chart will help you classify polygons.</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;"><i>Polygon</i></th> <th style="text-align: left;"><i>Sides</i></th> </tr> </thead> <tbody> <tr> <td>Triangle</td> <td>3</td> </tr> <tr> <td>Quadrilateral</td> <td>4</td> </tr> <tr> <td>Pentagon</td> <td>5</td> </tr> <tr> <td>Hexagon</td> <td>6</td> </tr> </tbody> </table>	<i>Polygon</i>	<i>Sides</i>	Triangle	3	Quadrilateral	4	Pentagon	5	Hexagon	6	<p>A parallelogram is a quadrilateral with</p> <ul style="list-style-type: none"> • opposite sides parallel • opposite sides congruent • opposite angles congruent • consecutive angles supplementary <div style="text-align: center;">  </div>
<i>Polygon</i>	<i>Sides</i>										
Triangle	3										
Quadrilateral	4										
Pentagon	5										
Hexagon	6										

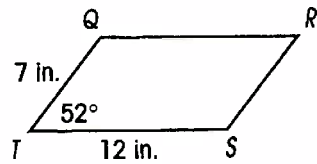
Classify each polygon. Then, name it.







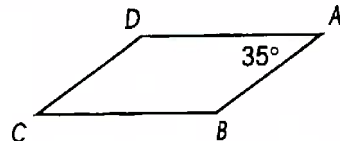
In the diagram, $QRST$ is a parallelogram. Find the length of each side and the measure of each angle.



- | | |
|--------------------|--------------------|
| 4. \overline{QR} | 5. \overline{RS} |
| 6. $\angle QRS$ | 7. \overline{TS} |
| 8. $\angle RST$ | 9. $\angle RQT$ |

CRITICAL THINKING

In parallelogram $ABCD$, $\angle A$ is 35° . What is the measure of $\angle B$, $\angle C$, and $\angle D$?



7 Special Parallelograms

Exercise 38

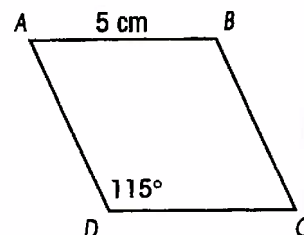
Lessons 7.3 and 7.4

A rhombus is a parallelogram with four congruent sides.
The diagonals of a rhombus

- bisect each other
- are perpendicular
- bisect each pair of opposite angles

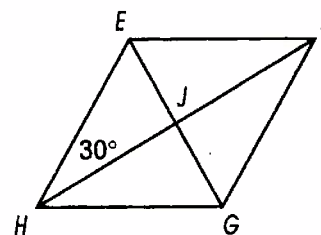
In the diagram, $ABCD$ is a rhombus. Find the length of each side and the measure of each angle.

1. \overline{AD}
2. $\angle B$
3. $\angle C$
4. $\angle A$
5. \overline{CD}
6. \overline{BC}



In the diagram, $EFGH$ is a rhombus. EJ is 13 inches. Find the length of each side and the measure of each angle.

7. \overline{JG}
8. \overline{EG}
9. $\angle HFG$
10. $\angle JHG$
11. $\angle EHG$
12. $\angle EJH$



CRITICAL THINKING

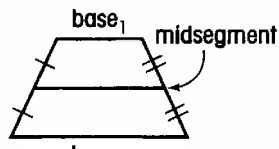
Parallelogram $LMNP$ is a rhombus with diagonals that are 12 mm and 16 mm long. What is the length of each side of the rhombus?

7 Trapezoids Exercise 39

Lessons 7.5 and 7.6

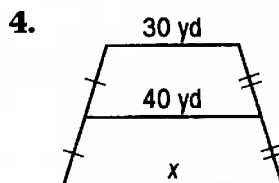
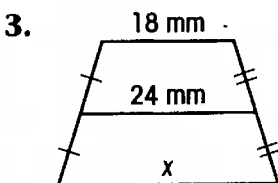
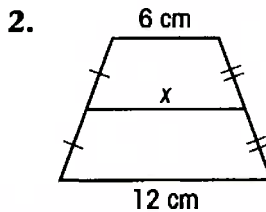
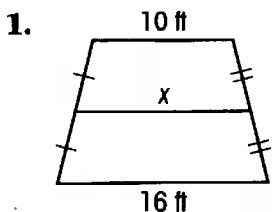
A trapezoid is a quadrilateral with only one pair of parallel sides.

An isosceles trapezoid has congruent legs. Each pair of base angles is congruent.



midsegment = $\frac{1}{2}(b_1 + b_2)$

Find the value of x in each trapezoid.



Trapezoid $STAR$ is isosceles. Find the length of each side and the measure of each angle.

5. $\angle S$

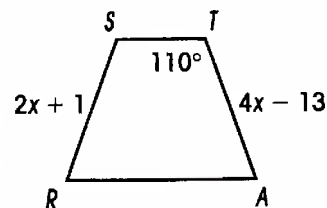
6. \overline{SR}

7. \overline{TA}

8. $\angle A$

9. $\angle R$

10. $\angle T$



CRITICAL THINKING

Can the two bases of a trapezoid be the same length? Explain your thinking.

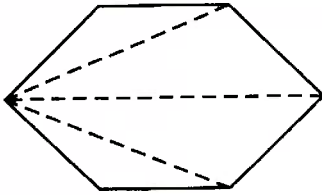
7 Problem-Solving Skill: Interior-Angle Sum of a Polygon

Exercise 40

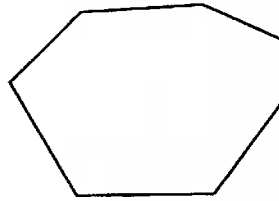
Lesson 7.8

Divide the polygon into triangles. Then, multiply the number of triangles by 180 to find the interior-angle sum for each polygon. The first one is started for you.

1.



2.



3. Complete the table below. Write the sum of the measures of the interior angles for each figure.

Name of Figure	Number of Sides (n)	Number of Triangles ($n - 2$)	Interior-Angle Sum ($n - 2$) • 180
Triangle	3	1	180
Quadrilateral	4	2	360
Pentagon	5	3	
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
n -gon	n		

Find the measure of an interior angle of each regular polygon.

4. pentagon

5. octagon

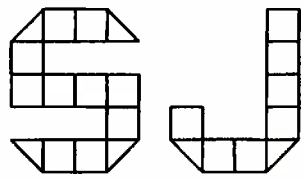
6. quadrilateral

7 Problem-Solving Application: Tiling a Surface **Exercise 41**

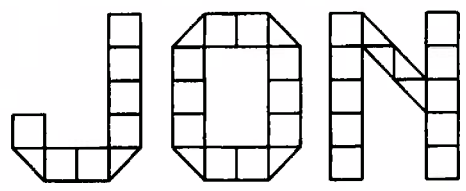
Lesson 7.9

Use the diagrams on the right to solve each problem.

- Susan Jones used this pattern to make a tile mosaic of her initials. How many square tiles and triangular tiles did she use?



- Jon used tiles to spell his name on his bedroom door. How many square tiles did he use? How many triangular tiles?



- Each tile measures 1 inch on a side. If Jon left $\frac{1}{4}$ inch of space between each letter, what was the length and width of his name?

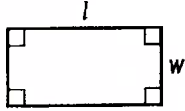
©Pearson Education, Inc./Globe Fearon, ©Pacemaker Geometry. All rights reserved.

8 Perimeter of Polygons

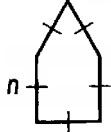
Exercise 42

Lesson 8.1

The perimeter of a polygon is the distance around the polygon.

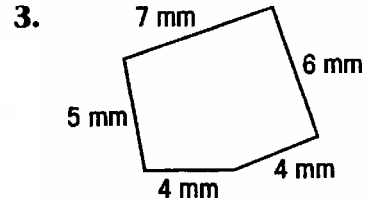
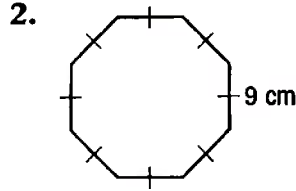
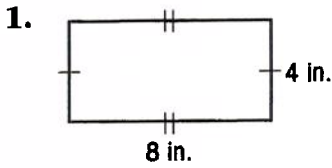


$P = 2w + 2l$

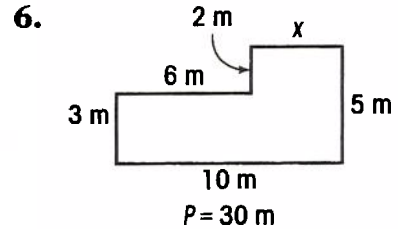
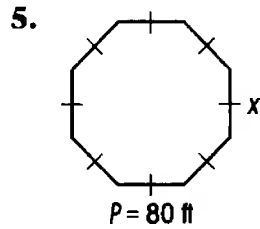
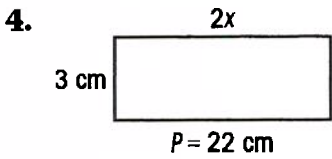


$P = 5n$

Find the perimeter of each polygon.

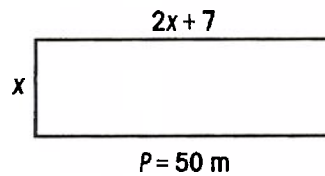


Find the value of x in each polygon.



CRITICAL THINKING

Find the value of x in the diagram. What is the length and width of this rectangle?



8 Area of Rectangles, Squares, and Parallelograms

Exercise 43

Lessons 8.2 and 8.3

The area of a polygon is the number of square units needed to cover the surface.

These formulas will help you find the area of polygons.

Polygon

Rectangle

Square

Parallelogram

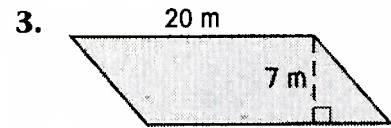
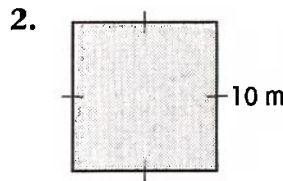
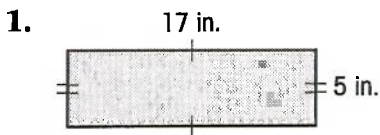
Formula

$$A = lw$$

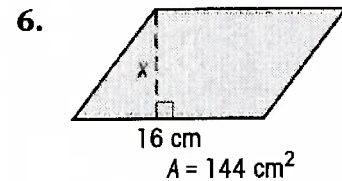
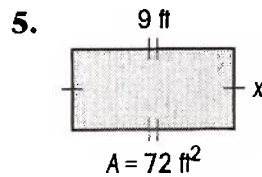
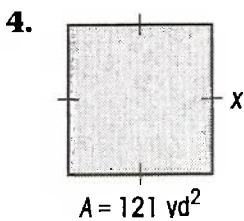
$$A = s^2$$

$$A = bh$$

Find the area of each figure.

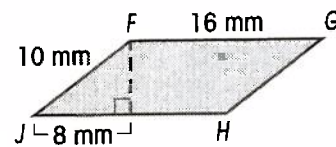


Find the value of x in each figure. The area is given.



CRITICAL THINKING

Find the area of $\square FGHI$. (Hint: Use the Pythagorean Theorem to find the height.)



8 Area of Triangles

Exercise 44

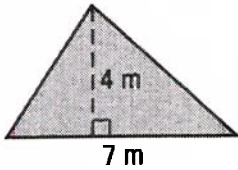
Lesson 8.4

Remember the formula for the area of a triangle.

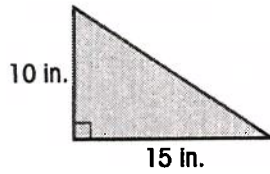
$$A = \frac{1}{2}bh$$

Find the area of each triangle.

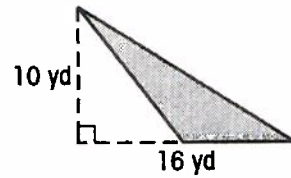
1.



2.

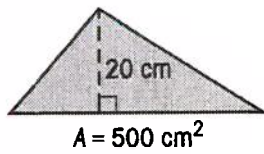


3.

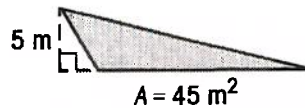


Find the length of each base. The area is given.

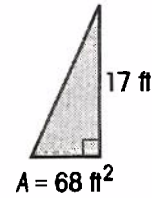
4.



5.

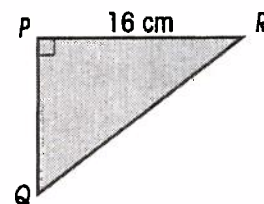


6.



CRITICAL THINKING

The area of $\triangle RPQ$ is 96 cm^2 . Find the length of \overline{PQ} and \overline{RQ} . (Hint: Once you find PQ , use the Pythagorean Theorem to find RQ .)



8 Area of Trapezoids**Exercise 45**

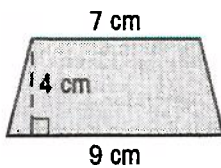
Lesson 8.5

Remember the formula for the area of a trapezoid.

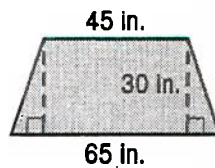
$$A = \frac{1}{2}h(b_1 + b_2)$$

Find the area of each trapezoid.

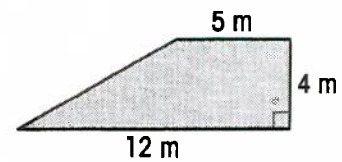
1.



2.



3.



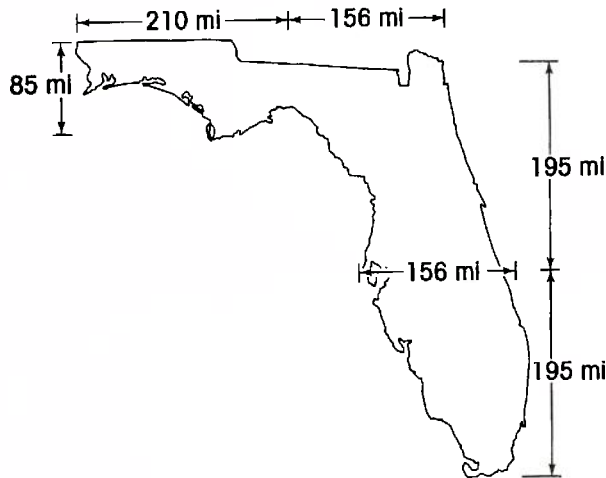
4. The area of a trapezoid is 140 m^2 . The bases are 18 m and 10 m. Find the height.
5. The area of a trapezoid is 100 cm^2 . The bases are 19 cm and 21 cm. Find the height.

CRITICAL THINKING

A trapezoid has an area of 120 ft^2 and a height of 20 ft. Find the possible pairs of integer bases. (Hint: One possible pair is 10 ft and 2 ft.)

**8 Problem-Solving Strategy:
Simplify the Problem****Exercise 46***Lesson 8.7*

Divide an irregular shape into polygons to estimate its area. Solve each problem using this map of Florida.

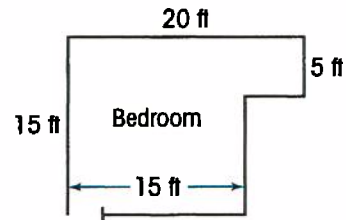


1. Estimate the area of Florida using two polygons.
2. Estimate the area of Florida using three or more polygons.
3. The actual area of Florida is $58,664 \text{ mi}^2$. Which estimate of the area is closer to the actual area of Florida? Explain your answer.

**8 Problem-Solving Application:
Carpeting an Area****Exercise 47***Lesson 8.8*

Use the diagram to solve each problem. Show your work.

1. How many square feet of carpeting does Dena need to carpet this room?



2. The carpet Dena wants to buy costs \$20 per square yard. How much will the carpet cost for this room?
(Hint: 1 square yard = 9 square feet)

3. Dena wants to put a pad under the carpet. It costs \$3 per square yard. How much will the pad cost?

9 Similar Triangles

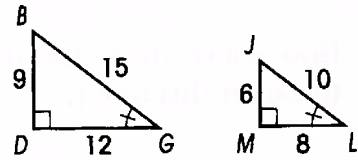
Exercise 48

Lessons 9.3 and 9.4

Similar triangles are the same shape, but may or may not be the same size.

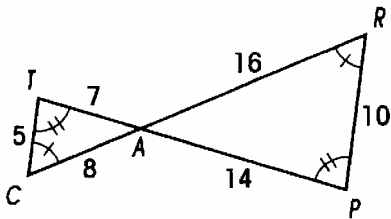
- Corresponding angles are congruent.
- The ratios of the lengths of the corresponding sides are equal. This is the similarity ratio.

$\triangle DBG \sim \triangle MJL$ The similarity ratio is 3 to 2.

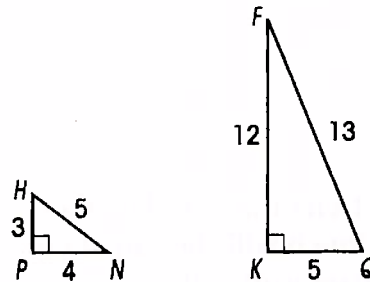


Decide if the two triangles are similar. Write *yes* or *no*.
If yes, write a similarity statement. Find the similarity ratio.

1.

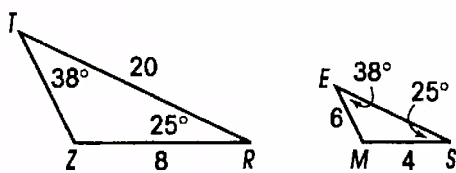


2.

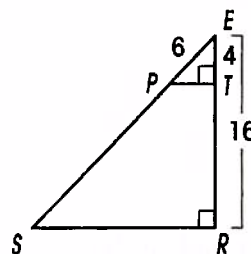


Each diagram shows similar triangles. Find *ES*.

3.



4.



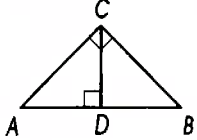
CRITICAL THINKING

$\triangle ABC \sim \triangle DEF$. The similarity ratio is 1 to 2.
 \overline{AB} is 6 cm long. How long is \overline{DE} ?

9 Altitude of a Right Triangle Exercise 49

Lesson 9.5

The altitude from the right angle of a right triangle forms three similar triangles.

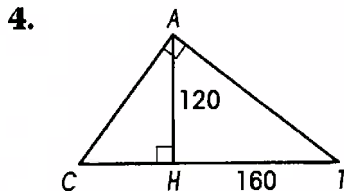
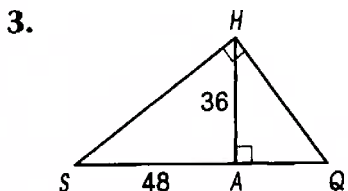
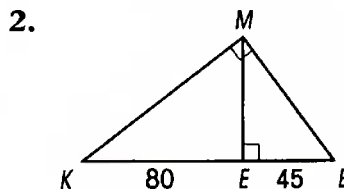
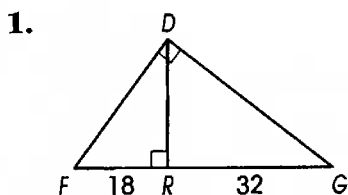


The altitude is the geometric mean between the parts of the base.

$$\triangle ABC \sim \triangle CAD \sim \triangle CBD$$

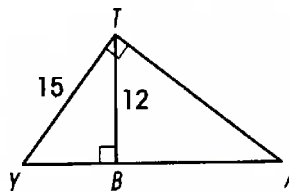
$$\frac{AD}{CD} = \frac{CD}{DB}$$

Find all three unknown lengths in each triangle.



CRITICAL THINKING

Find all three unknown lengths in the diagram on the right.



9 Legs of a Right Triangle

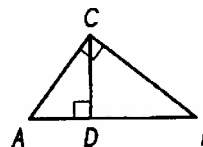
Exercise 50

Lesson 9.6

Each leg of a right triangle is a geometric mean between part of the base and the whole base.

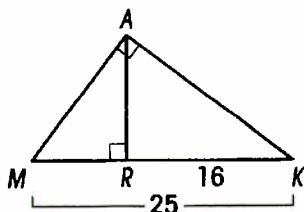
For the diagram on the right,

$$\frac{AD}{AC} = \frac{AC}{AB} \text{ and } \frac{BD}{BC} = \frac{BC}{AB}$$

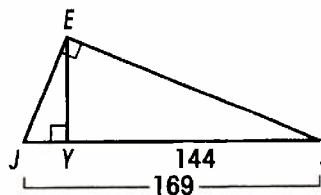


Find the length of each leg in each large triangle.

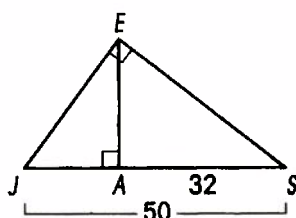
1.



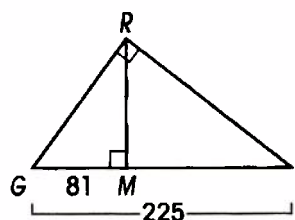
2.



3.

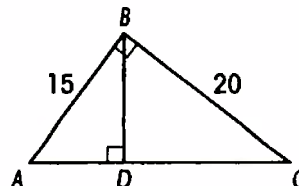


4.



CRITICAL THINKING

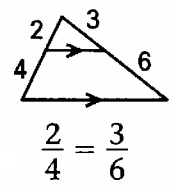
Find all three unknown lengths in the diagram on the right.



9 Side-Splitter Theorem Exercise 51

Lesson 9.7

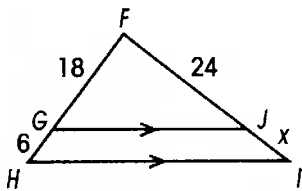
Any line that connects two sides of a triangle and is parallel to the third side divides the two sides proportionally.



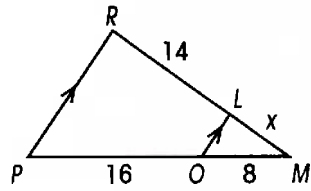
$$\frac{2}{4} = \frac{3}{6}$$

Find the value of x in each triangle.

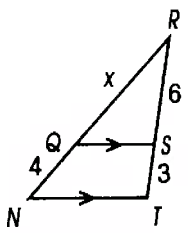
1.



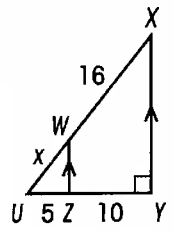
2.



3.

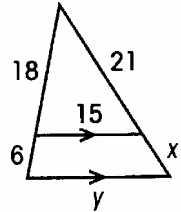


4.



CRITICAL THINKING

Solve for x and y . Explain how you found the value of y .



9 Similar Polygons

Exercise 52

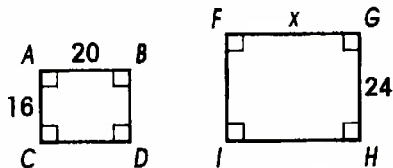
Lessons 9.8 and 9.9

Similar polygons have congruent corresponding angles and proportional corresponding sides. The ratio of their perimeters is equal to the ratio of any pair of corresponding sides.

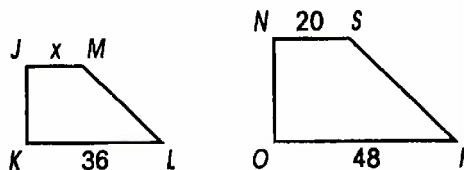
$$\frac{\text{Perimeter}_1}{\text{Perimeter}_2} = \frac{\text{side}_1}{\text{side}_2}$$

Each pair of polygons is similar. Find the value of x . Write the similarity ratio.

1.

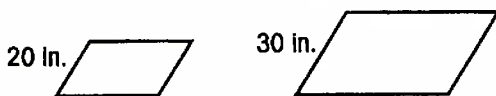


2.



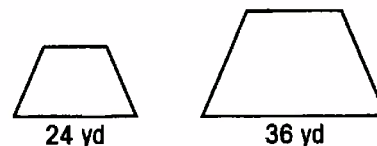
Each pair of polygons is similar. The measures of corresponding sides are given. Find the unknown perimeter.

3.



$P = 80 \text{ in.}$ $P = \underline{\hspace{2cm}}$

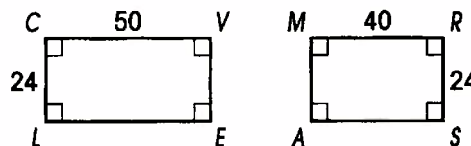
4.



$P = \underline{\hspace{2cm}}$ $P = 90 \text{ yd}$

CRITICAL THINKING

Explain why this pair of rectangles is not similar.



9 Area of Similar Polygons

Exercise 53

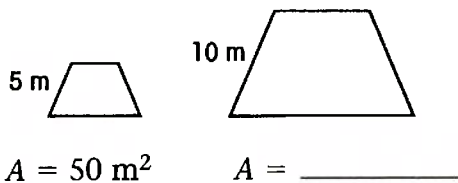
Lesson 9.10

If two polygons are similar, the ratio of their areas is equal to the ratio of the squares of the lengths of any pair of corresponding sides.

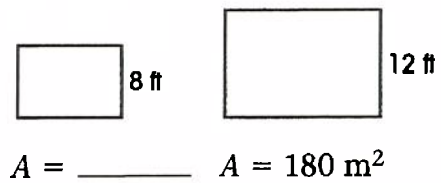
$$\frac{\text{Area}_1}{\text{Area}_2} = \frac{(\text{side}_1)^2}{(\text{side}_2)^2}$$

Each pair of polygons is similar. The measures of corresponding sides are given. Find the unknown area.

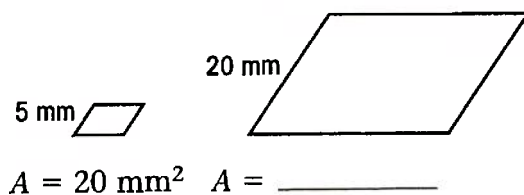
1.



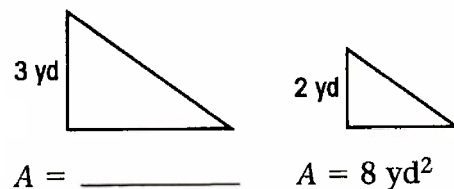
2.



3.



4.



CRITICAL THINKING

$\triangle STU \sim \triangle JMC$ The area of $\triangle STU$ is 36 in.^2 The area of $\triangle JMC$ is 144 in.^2 What is the similarity ratio?

**9 Problem-Solving Strategy:
Write an Equation****Exercise 54***Lesson 9.12***Solve each problem. Show your work.**

1. A 5-ft-tall boy casts a shadow that is 4 ft long. At the same time, a big tree casts a shadow that is 16 ft long. What is the height of the tree?

2. A 2-ft-tall child casts a shadow that is 8 ft long. At the same time, a tree casts a shadow 32 ft long. How tall is the tree?

3. A 6-ft-tall man casts a shadow that is about 10 ft long. At the same time, a building casts a shadow 60 ft long. How tall is the building?

4. A tower casts a shadow that is about 210 m long. At the same time, a 4-m post casts a shadow 8 m long. What is the height of the tower?

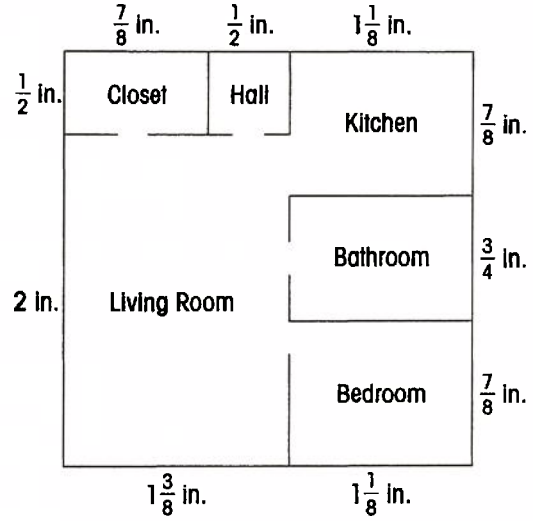
9 Problem-Solving Application: Scale Drawings **Exercise 55**

Lesson 9.13

Use the scale drawing to solve each problem.
Show your work.

1. What is the length of the kitchen?

2. What is the width of the kitchen?



Scale: $\frac{1}{8}$ inch = 1 foot

3. What is the length of the closet?

4. What is the width of the closet?

5. What is the area of the apartment? (Hint: Find the length and width.)

10 Circumference and Area of a Circle

Exercise 56

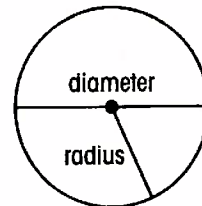
Lessons 10.1 and 10.2

To find the perimeter and area of a circle, use these formulas.

Circumference = $\pi \cdot \text{diameter}$ or $C = \pi d$

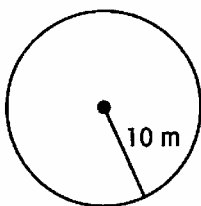
Circumference = $\pi \cdot 2 \cdot \text{radius}$ or $C = 2\pi r$

Area = $\pi \cdot (\text{radius})^2$ or $A = \pi r^2$

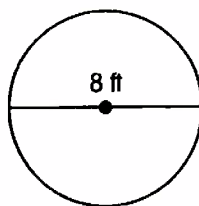


Find the circumference of each circle. Use 3.14 for π in exercises 1–2. Use $\frac{22}{7}$ for π in exercise 3.

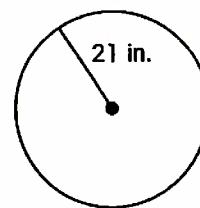
1.



2.

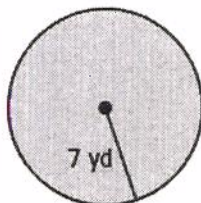


3.

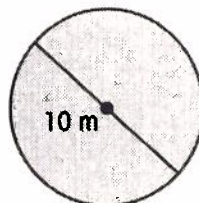


Find the area of each circle. Use 3.14 for π .

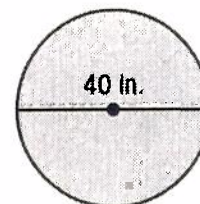
4.



5.



6.



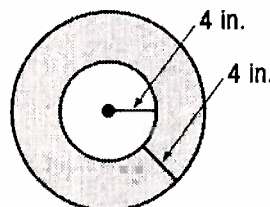
Find the length of the radius using the given information.

7. $C = 10\pi$ cm

8. $A = 225\pi$ m²

CRITICAL THINKING

Look at the diagram on the right.
Find the area of the shaded part.



10 Arcs, Central Angles, and Sectors

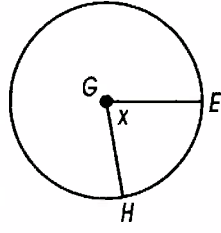
Exercise 57

Lessons 10.3 and 10.4

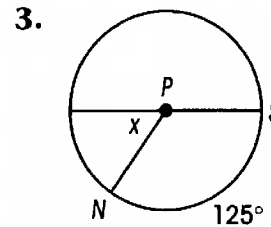
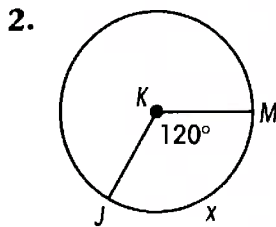
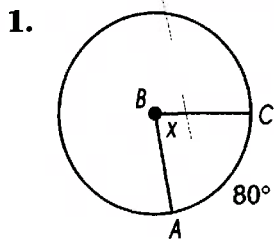
The measure of a central angle is equal to the degree measure of its arc.
 $m\angle EGH = m\widehat{EH}$

To find the length of an arc, use this formula.
 length of $\widehat{EH} = \frac{m\angle x}{360} \cdot 2\pi r$

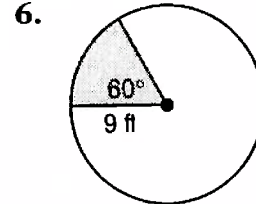
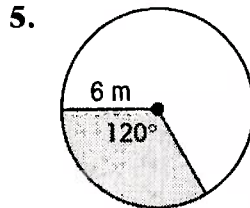
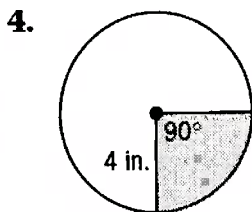
To find the area of a sector, use this formula.
 Area of sector $EGH = \frac{m\angle x}{360} \cdot \pi r^2$



Find the value of x in each circle.



Find the length of each minor arc. Find the area of each sector. Use 3.14 for π .



CRITICAL THINKING

The area of a sector of a circle is 31.4 cm^2 , and the radius is 6 cm. Find the measure of the central angle. Use 3.14 for π .

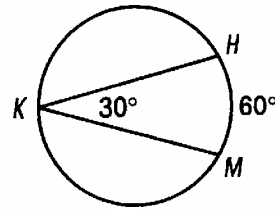
10 Inscribed Angles

Exercise 58

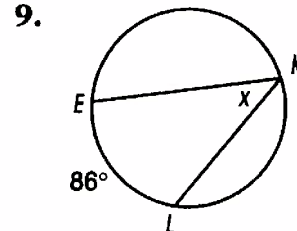
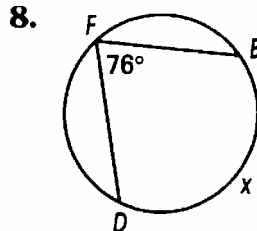
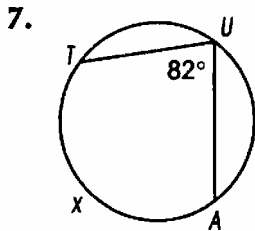
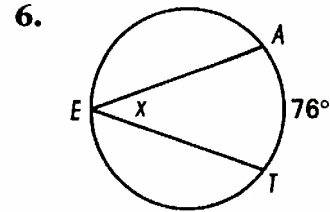
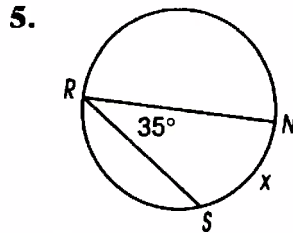
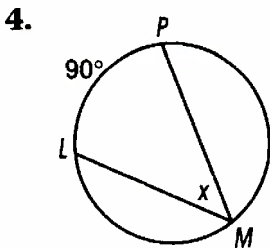
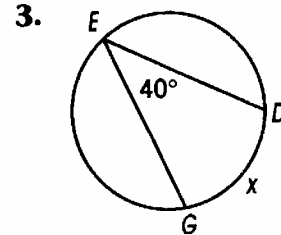
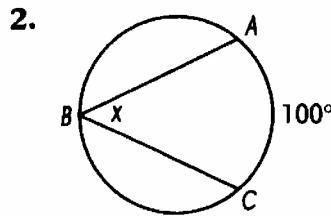
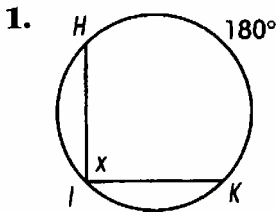
Lesson 10.5

The measure of an inscribed angle is equal to one-half the degree measure of its arc.

$$m\angle HKM = \frac{1}{2}m\widehat{HM}$$



Find the value of x in each circle.



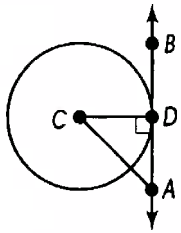
CRITICAL THINKING

In $\odot C$, the measure of the central angle ACD is 88° . What is the measure of the inscribed angle AED that cuts arc \widehat{AD} ? (Hint: Draw a diagram. Then, find the measure of the arc.)

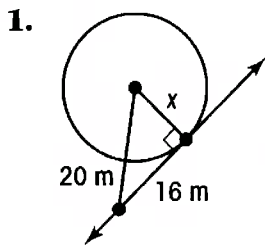
10 Tangents **Exercise 59**

Lesson 10.6

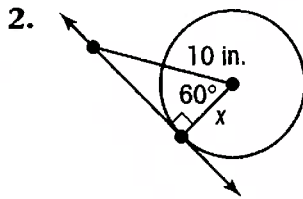
\overleftrightarrow{AB} is tangent to $\odot C$ at point D .
 \overleftrightarrow{AB} is perpendicular to the radius, \overline{CD} .
 $\triangle ADC$ is a right triangle.



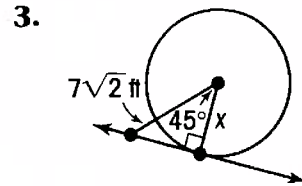
The given line is tangent to each circle. Find the value of x for each circle. Some formulas you can use are given.



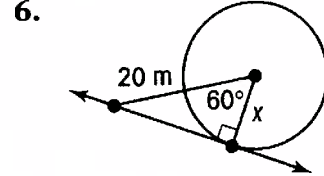
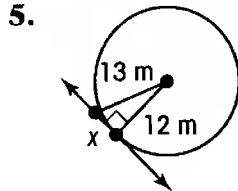
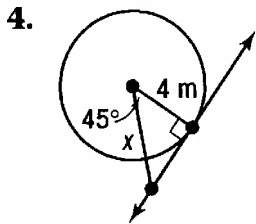
$a^2 + b^2 = c^2$



hypotenuse = 2 • short leg

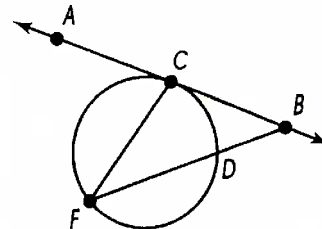


hypotenuse = leg $\sqrt{2}$



CRITICAL THINKING

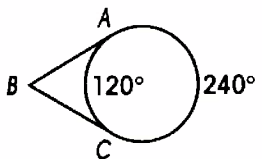
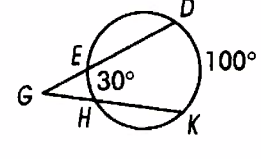
\overleftrightarrow{AB} is tangent to this circle at point C . $m\widehat{CD}$ is 60° .
 \overline{CF} is a diameter. Find the measure of $\angle CBF$.



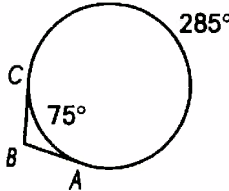
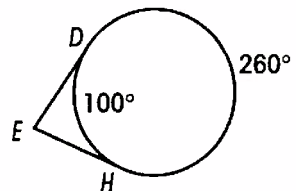
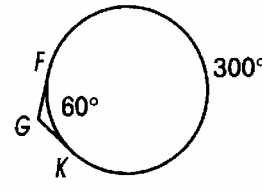
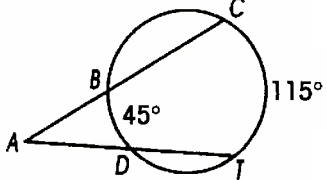
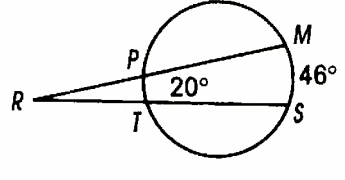
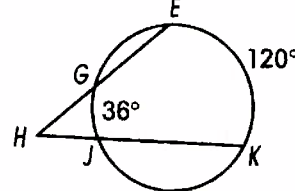
10 Tangents, Secants, and Angles

Exercise 60

Lesson 10.7

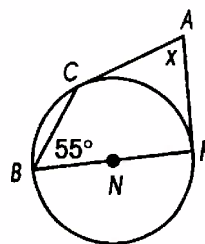
Tangent Angle	Secant Angle
	
$m\angle B = \frac{1}{2}(240 - 120) = 60$	$m\angle G = \frac{1}{2}(100 - 30) = 35$

Find the measure of the tangent angle or secant angle for each circle.

1. 
2. 
3. 
4. 
5. 
6. 

CRITICAL THINKING

In $\odot N$, $\angle PBC$ is an inscribed angle. Find the degree measure of \widehat{PC} and \widehat{PBC} . Then, find the value of x .



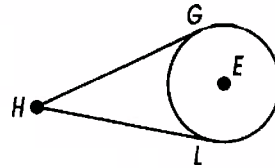
10 Tangents and Segments

Exercise 61

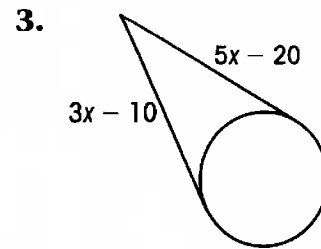
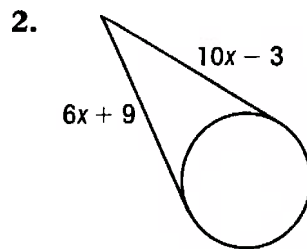
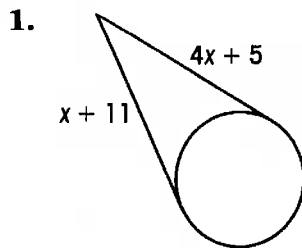
Lesson 10.8

Two tangents from the same point outside a circle are congruent.

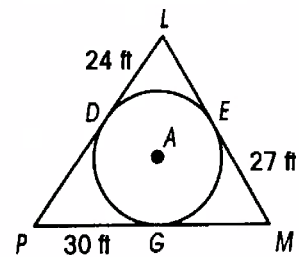
In $\odot E$, $\overline{HG} \cong \overline{HL}$.



In each diagram, the line segments are tangent to the circle. Find the value of x .

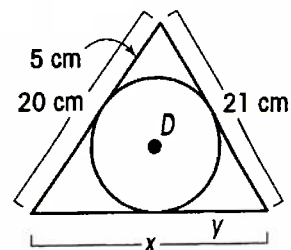


4. In the diagram, each line segment is tangent to $\odot A$. Find the perimeter of $\triangle PLM$.



CRITICAL THINKING

The given line segments are tangent to $\odot D$. Find the values of x and y .



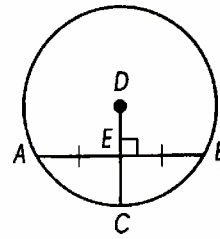
10 Chords

Exercise 62

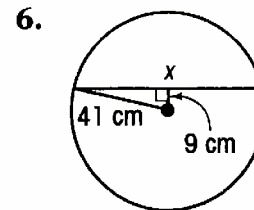
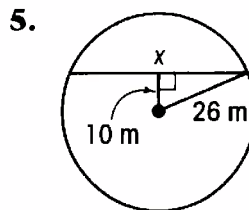
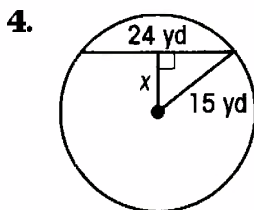
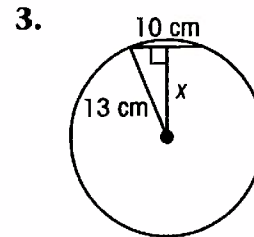
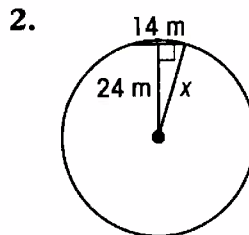
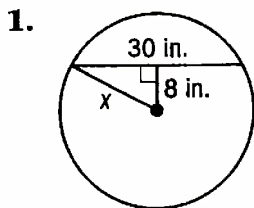
Lesson 10.9

\overline{AB} is a chord of $\odot D$. Any line segment from the center of the circle that is perpendicular to a chord bisects the chord.

$$AE = EB$$



Find the value of x in each circle.



CRITICAL THINKING

The diameter of $\odot B$ is 30 m. Chord DE is 18 m. How far is \overline{DE} from the center of $\odot B$? (Hint: Draw a diagram.)

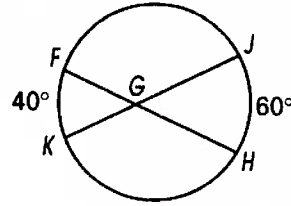
10 Chords and Angles

Exercise 63

Lesson 10.10

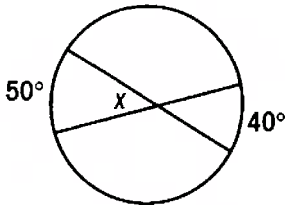
The measure of an angle formed by two intersecting chords is one-half the sum of the intercepted arcs.

$$m\angle FGK = \frac{1}{2}(40 + 60) = 50$$

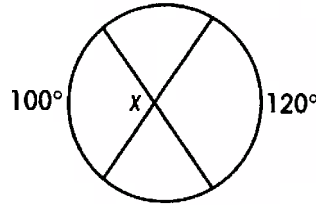


Find the value of x in each circle.

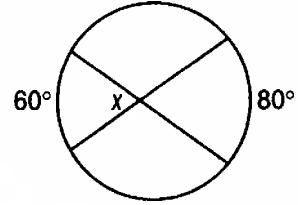
1.



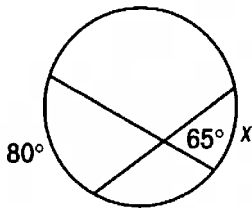
2.



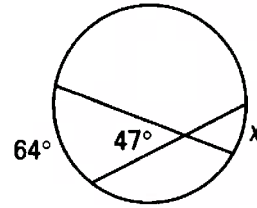
3.



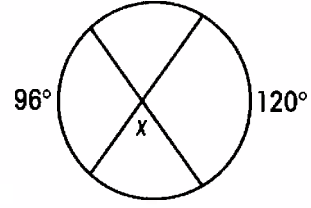
4.



5.

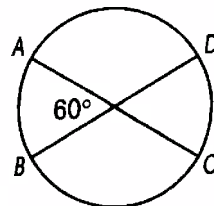


6.



CRITICAL THINKING

Find one possible degree measure for each of the intercepted arcs, \widehat{AB} and \widehat{CD} , in the circle on the right.



10 Chords and Segments

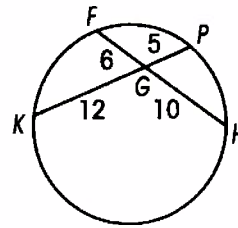
Exercise 64

Lesson 10.11

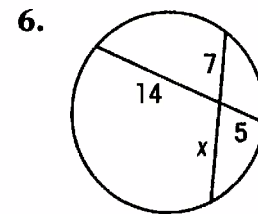
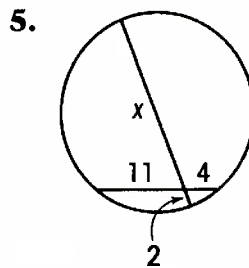
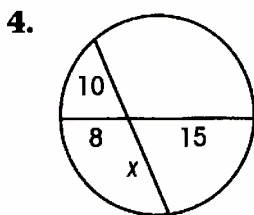
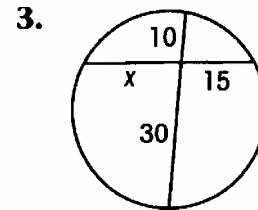
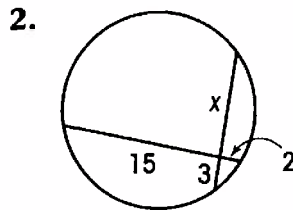
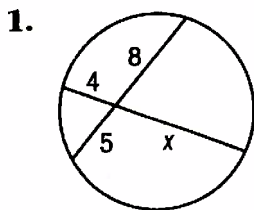
When two chords intersect, the segments have a special relationship.

$$FG \cdot GH = PG \cdot GK$$

$$6 \cdot 10 = 5 \cdot 12$$

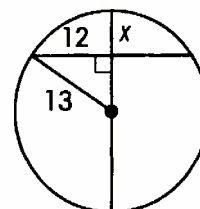


Find the value of x in each circle.



CRITICAL THINKING

Find the value of x in the diagram on the right.



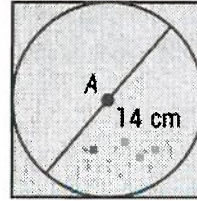
10 Problem-Solving Skill: Inscribed and Circumscribed Circles

Exercise 65

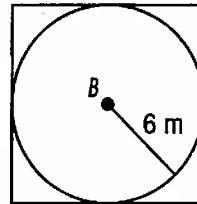
Lesson 10.13

Solve each problem. Show your work.

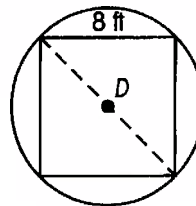
1. $\odot A$ is inscribed in a square.
Find the area of the square.



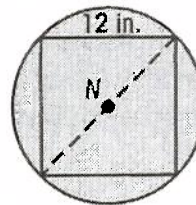
2. $\odot B$ is inscribed in a square.
Find the perimeter of the square.



3. $\odot D$ is circumscribed about a square. Find the circumference of $\odot D$. Leave π in your answer.



4. $\odot N$ is circumscribed about a square. Find the area of $\odot N$. Leave π in your answer.



10 **Problem-Solving Application:**
Revolutions of a Circle**Exercise 66***Lesson 10.14***Solve each problem. Show your work.**

1. Roy rides a bicycle with 24-inch wheels. What is the distance in feet he can travel in 5,000 revolutions?

2. Marjorie rides a bicycle with 26-inch wheels. She travels 5,200 feet. About how many revolutions of the wheel is that?

3. Jacob rides a bicycle with 30-inch wheels. He can travel 150 revolutions in one minute. At this rate, what is the distance in feet he can travel in 5 minutes?

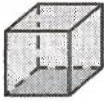
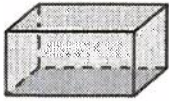
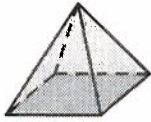
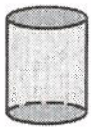


4. Inez rides a bicycle with 24-inch wheels. Barry rides a bicycle with 26-inch wheels. What is the distance in feet each person can travel in 1,500 revolutions? How much farther can Barry travel?

11 Space Figures

Exercise 67

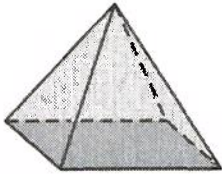
Lessons 11.1 and 11.2

Space figures are three-dimensional solid shapes.

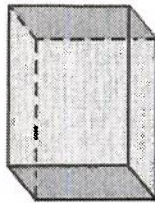
<p>Cube</p> 	<p>Rectangular Prism</p> 	<p>Square Pyramid</p> 
<p>Cylinder</p> 	<p>Cone</p> 	<p>Sphere</p> 

Name each space figure.

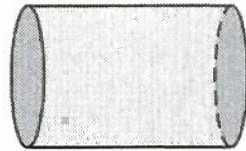
1.



2.

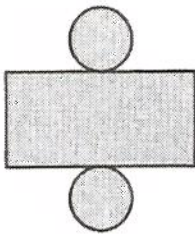


3.

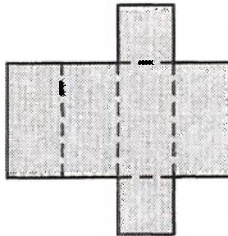


Name the space figure you can make from each net.

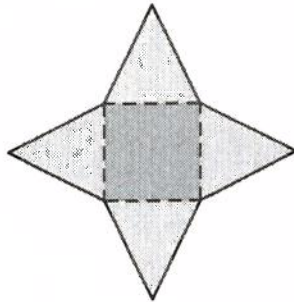
4.



5.

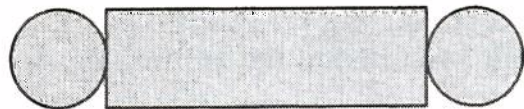


6.



CRITICAL THINKING

Can this diagram be a net for a cylinder?



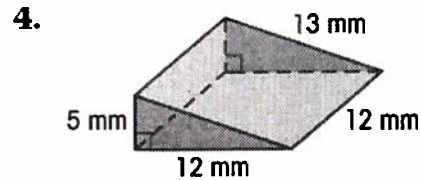
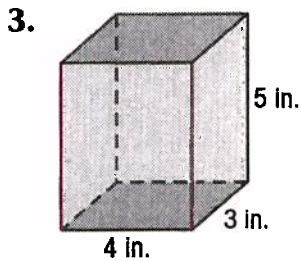
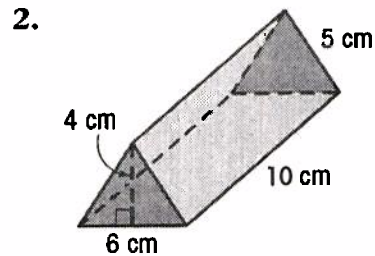
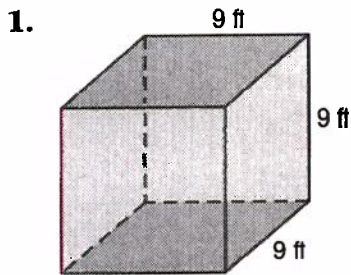
11 Surface Area of a Prism

Exercise 68

Lesson 11.3

The surface area of a prism is the sum of the areas of the faces. Using nets can help you find surface area.

Find the surface area of each prism.

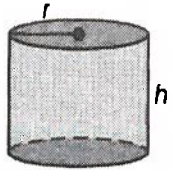



CRITICAL THINKING

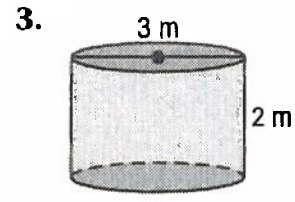
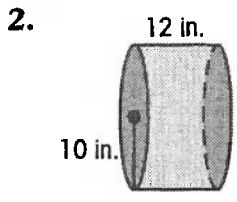
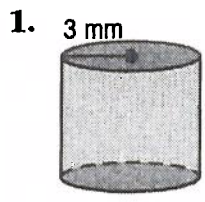
Can you find the surface area of a prism if you know that it is a cube and that the width is 2 ft? Explain why or why not. If you can find the surface area, what is it?

11 Surface Area of a Cylinder and a Sphere Exercise 69

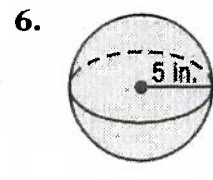
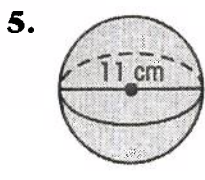
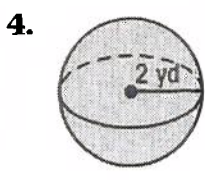
Lessons 11.4 and 11.5

<p>Cylinder</p> <p>Surface Area = $2\pi(\text{radius})^2 + (2\pi \text{ radius}) \text{ height}$ $SA = 2\pi r^2 + 2\pi rh$</p> 	<p>Sphere</p> <p>Surface Area = $4\pi(\text{radius})^2$ $SA = 4\pi r^2$</p> 
--	---

Find the surface area of each cylinder to the nearest square unit. Use 3.14 for π .



Find the surface area of each sphere. Write your answer in terms of π .



Find the radius of each sphere. You are given the surface area.

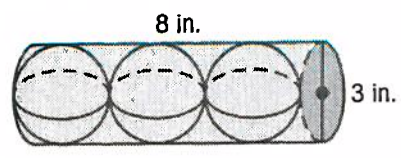
7. $SA = 100\pi \text{ m}^2$

8. $SA = 676\pi \text{ in.}^2$

9. $SA = 144\pi \text{ cm}^2$

CRITICAL THINKING

Three tennis balls fit in the container on the right. What is the total surface area of the tennis balls? What is the surface area of the container? Write your answer in terms of π .



11 Volume of a Prism**Exercise 70**

Lesson 11.6

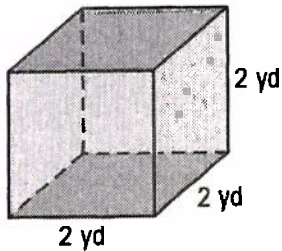
The volume of a prism is the number of cubic units that fill it.

Volume of a prism = Area of base • height

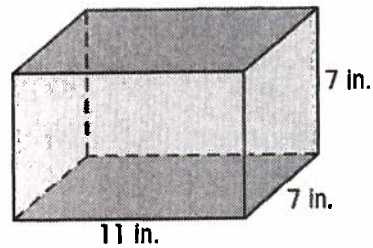
$$V = Bh$$

Find the volume of each prism.

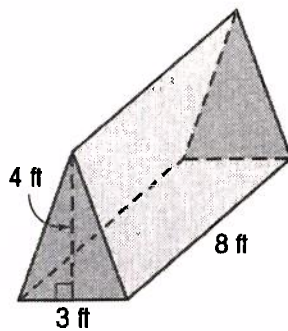
1.



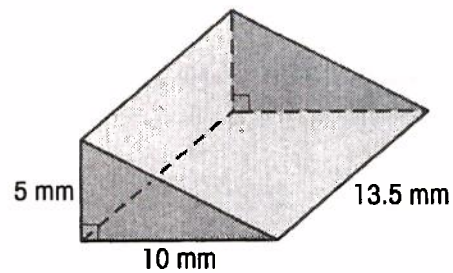
2.



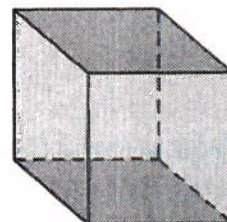
3.



4.

**CRITICAL THINKING**

The cube on the right has a volume of 125 in.^3 . What is the length of one side of the cube? Explain how you found your answer.

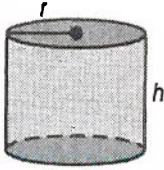


11 Volume of a Cylinder **Exercise 71**

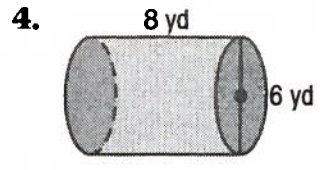
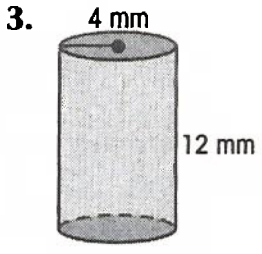
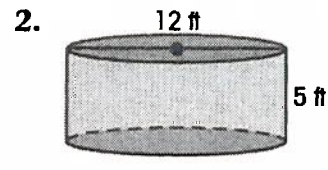
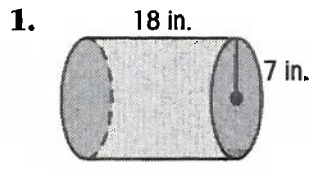
Lesson 11.7

Cylinder

Volume of a cylinder = Area of base • height

$$V = \pi r^2 h$$


Find the volume of each cylinder. Write your answer in terms of π .



Find the height of each cylinder. You are given the volume and the radius of the base.

- | | | |
|---|--|---|
| 5. $V = 864\pi \text{ in.}^3$
$r = 12 \text{ in.}$ | 6. $V = 250\pi \text{ m}^3$
$r = 5 \text{ m}$ | 7. $V = 6,000\pi \text{ ft}^3$
$r = 10 \text{ ft}$ |
|---|--|---|

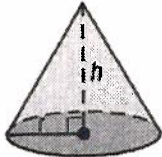

CRITICAL THINKING

Describe what happens to the volume of a cylinder when you multiply its radius by 10. Also, describe what happens to the volume of a cylinder when you multiply its height by 10.
(Hint: Use a cylinder with $r = 2$ and $h = 3$ to check your answer.)

11 Volume of a Cone and a Sphere

Exercise 72

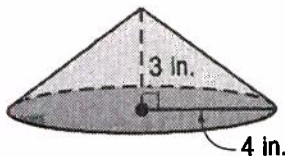
Lessons 11.8 and 11.9

Cone	Sphere
$\text{Volume} = \frac{1}{3}\pi(\text{radius})^2 \cdot (\text{height})$ $V = \frac{1}{3}\pi r^2 h$	$\text{Volume of a sphere} = \frac{4}{3}\pi(\text{radius})^3$ $V = \frac{4}{3}\pi r^3$
	

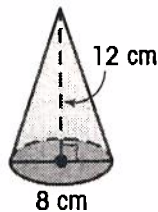
Find the volume of each cone to the nearest cubic unit.

Use 3.14 for π .

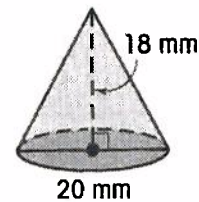
1.



2.



3.



Find the height of each cone.

4. $V = 18\pi \text{ in.}^3$
 $r = 3 \text{ in.}$

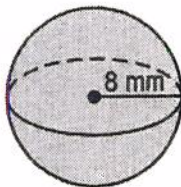
5. $V = 72\pi \text{ cm}^3$
 $r = 6 \text{ cm}$

6. $V = 300\pi \text{ m}^3$
 $r = 15 \text{ m}$

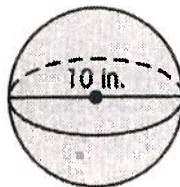
Find the volume of each sphere to the nearest cubic unit.

Use 3.14 for π .

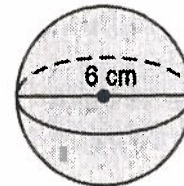
7.



8.



9.



CRITICAL THINKING

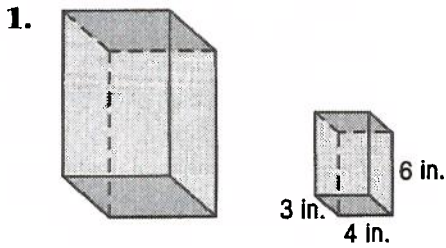
Walter has a solid brass cone with a radius of 10 cm and a height of 10 cm. Theresa has a solid brass sphere with a radius of 10 cm. Which has the greater volume? Explain.

11 Volume of Similar Figures **Exercise 73**

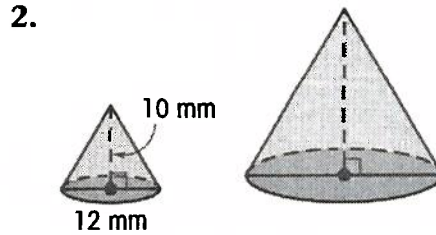
Lesson 11.10

If two figures are similar and the similarity ratio of the sides is $a:b$, then the ratio of the areas is $a^2:b^2$ and the ratio of the volumes is $a^3:b^3$.

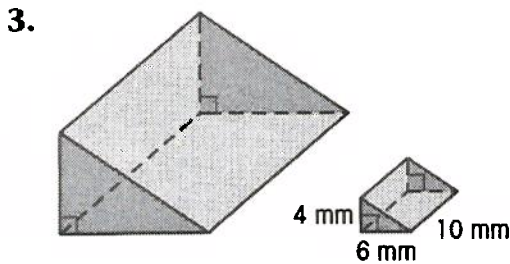
Each pair of figures is similar. Find the volume of the larger figure. Express your answer in terms of π for exercise 2.



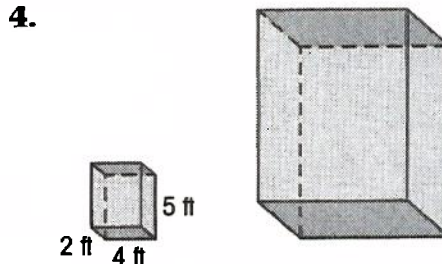
similarity ratio of 2:1



similarity ratio of 1:2



similarity ratio of 3:1



similarity ratio of 1:3

CRITICAL THINKING

The volume of a small sphere is $36\pi \text{ cm}^3$. The volume of a large sphere is $288\pi \text{ cm}^3$ and its radius is 6 cm. What is the similarity ratio of the radii of the two spheres?

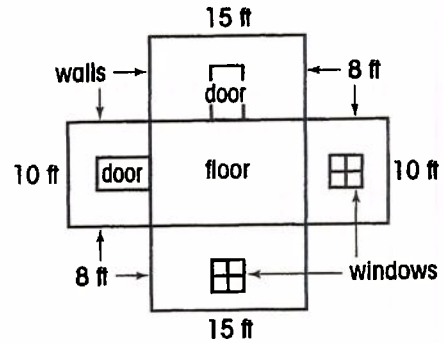
11 Problem-Solving Strategy: Write an Equation

Exercise 74

Lesson 11.12

Write an equation to solve each problem.

- Jeremy wants to paint a room that measures 10 ft by 15 ft and has 8-ft-high walls. The room has two doors that are each 4 ft by 7 ft and two windows that are each 3 ft by 3 ft. How many square feet of walls will Jeremy paint?



- Perry's Pool Company is refinishing the inside walls and bottom of a swimming pool. The bottom of the pool is 50 m by 10 m. The pool is 3 m deep. How many square meters will be refinished? (Hint: Make a net.)
- Jonathan uses two coats of paint to cover the walls of his garage, including the door. The surface area of the walls and door is 608 ft^2 . A gallon of paint covers 350 ft^2 . The paint is only sold in gallon containers. How many gallons of paint should Jonathan buy?

**11 Problem-Solving Application:
Air Conditioning****Exercise 75***Lesson 11.13*

Solve each problem. Use the formula $BTU = 3(\text{Volume of room})$.
Use the table if necessary.

1. Jessica wants to buy an air conditioner for her bedroom. The room measures 12.5 ft by 15 ft and has an 8-ft-high ceiling. How many BTUs are needed to cool this room?

Model	BTU	Price
A	5,000	\$199.99
B	6,000	\$229.99
C	8,000	\$299.99
D	10,000	\$349.99
E	12,000	\$399.99

2. Use the table above to find which model air conditioner would cool a conference room that has 256 ft² of floor space and an 8-ft-high ceiling.
3. Andy is buying an air conditioner for two connecting rooms. One room measures 12 ft by 20 ft, and the other measures 12 ft by 15 ft. Both have 8-ft-high ceilings. How much will an air conditioner cost that cools both rooms?

12 Points on the Coordinate Plane and Finding Distance Exercise 76

Lessons 12.1 and 12.2

To locate or graph a point (x, y) , begin at the origin.

If the x -coordinate is positive, move to the right.
 If the x -coordinate is negative, move to the left.
 Then, if the y -coordinate is positive, move up.
 Then, if the y -coordinate is negative, move down.

To find the distance between two points (x_1, y_1) and (x_2, y_2) , use the distance formula.

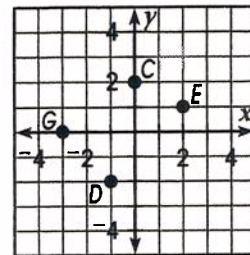
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Draw a coordinate plane on grid paper. Graph each point.

1. $H(0, -1)$ 2. $J(1, 0)$ 3. $A(-2, 1)$ 4. $B(1, -2)$

Write the coordinates of each point.

5. C _____ 6. D _____
 7. E _____ 8. G _____



Find the distance between each set of points.

9. $K(1, 2)$ and $T(0, 3)$ 10. $M(-3, 8)$ and $P(-9, 0)$
 11. $N(4, 3)$ and $L(1, -1)$ 12. $S(2, 3)$ and $R(-2, -3)$

CRITICAL THINKING

Points $A(2, 2)$, $B(2, -3)$, $C(-3, -3)$, and D form a square. Write the coordinates of point D .

12 Midpoint of a Line Segment and Slope of a Line

Exercise 77

Lessons 12.3 and 12.4

To find the midpoint of a line segment, use the midpoint formula.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

To find the slope of a line, use the slope formula.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

You are given the endpoints of a line segment. Find each midpoint.

1. A (6, 7) and B (4, 5)

2. C (-3, 2) and D (7, 4)

3. E (-3, -4) and G (3, 2)

4. H (0, 5) and J (-4, 7)

Find the slope of each line that contains the given points.

5. K (4, 5) and L (6, 7)

6. M (5, 2) and P (3, 5)

7. R (-2, 2) and S (2, 5)

8. T (-1, 4) and V (-2, 3)

9. A (0, 8) and C (-5, 4)

10. B (4, 2) and D (-4, -6)

CRITICAL THINKING

Points A (2, 2), B (1, 1), C (0, 0), and D (-1, -1) are on line m . Find the slope of line m using any two of these points. Now, find the slope of line m using another two of these points. Are the slopes the same? Why? Explain your thinking.

12 Parallel and Perpendicular Lines**Exercise 78**

Lesson 12.5

Lines are parallel if their slopes are equal.

Lines are perpendicular if their slopes are negative reciprocals of each other.

$\frac{3}{4}$ and $-\frac{4}{3}$ are negative reciprocals. -4 and $\frac{1}{4}$ are negative reciprocals.

Decide if the lines are parallel. Write *parallel* or *not parallel*.

- Line 1 points: (2, 4) and (3, 1)
Line 2 points: (4, 2) and (3, 5)
- Line 1 points: (-5, 4) and (2, 2)
Line 2 points: (3, -5) and (6, 4)
- Line 1 points: (1, 2) and (-3, 2)
Line 2 points: (3, 4) and (-1, 4)
- Line 1 points: (-5, 1) and (-2, 3)
Line 2 points: (5, -1) and (2, 1)

Decide if the lines are perpendicular. Write *perpendicular* or *not perpendicular*.

- Line 1 points: (0, 0) and (3, 2)
Line 2 points: (4, 1) and (2, 4)
- Line 1 points: (3, -2) and (1, 1)
Line 2 points: (-2, 1) and (4, 5)
- Line 1 points: (-3, 4) and (-1, 0)
Line 2 points: (-1, 0) and (5, 3)
- Line 1 points: (0, 4) and (-2, 5)
Line 2 points: (0, -1) and (3, 5)

CRITICAL THINKING

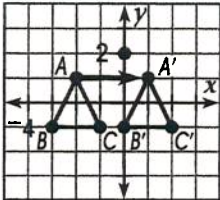
You are given the coordinates of the vertices of a parallelogram. How can you decide if the parallelogram is a square?

12 Translations in the Coordinate Plane Exercise 79

Lesson 12.6

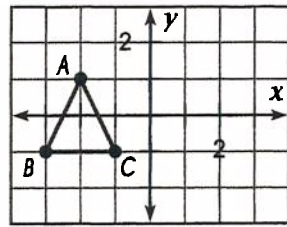
A translation, or slide, is a type of transformation. $\triangle ABC$ is the preimage. $\triangle A'B'C'$ is the image.

The rule for the transformation on the right is

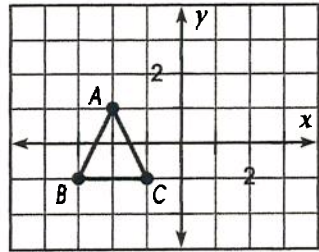
$$T(x, y) \rightarrow (x + 3, y)$$


Use each rule to translate $\triangle ABC$. Draw each translation image on the given coordinate plane.

1. $T(x, y) \rightarrow (x + 2, y - 1)$

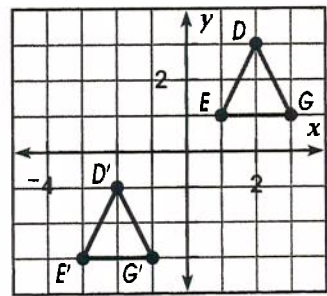


2. $T(x, y) \rightarrow (x - 1, y + 2)$



Use the diagram on the right.

3. Write the rule to describe the translation $\triangle DEG \rightarrow \triangle D'E'G'$.



CRITICAL THINKING

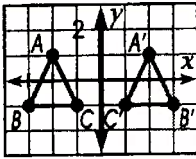
Points $A(0, 1)$, $B(-1, -1)$, and $C(1, -1)$ are the vertices of $\triangle ABC$. After a translation, $\triangle A'B'C'$ has coordinates $A'(1, -1)$, $B'(0, -3)$, and $C'(2, -3)$. Write the rule to describe the translation $\triangle ABC \rightarrow \triangle A'B'C'$.

12 Reflections in the Coordinate Plane Exercise 80

Lesson 12.7

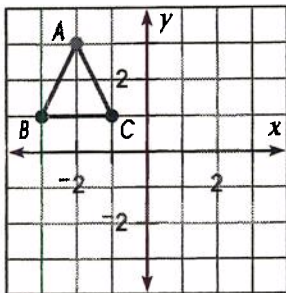
A reflection, or flip, is a type of transformation.

The preimage is reflected over a line of reflection. $\triangle ABC$ is reflected over the y -axis. $\triangle A'B'C'$ is the reflection image.

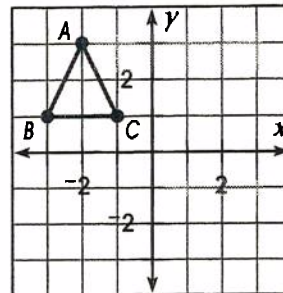


Reflect $\triangle ABC$ over each given line of reflection. Draw each reflection image.

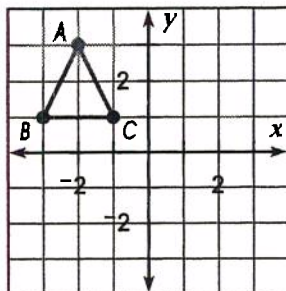
1. y -axis



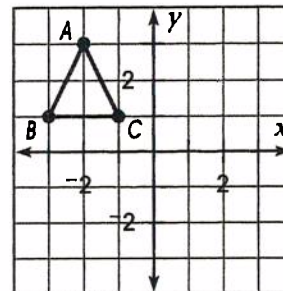
2. x -axis



3. $x = -1$



4. $y = 1$



CRITICAL THINKING

Reflect the reflection image $\triangle A'B'C'$ from exercise 2 over the y -axis. In which quadrant is the final reflection?

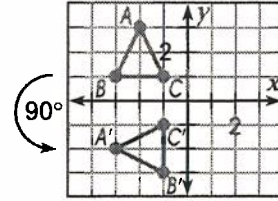
12 Rotations in the Coordinate Plane

Exercise 81

Lesson 12.8

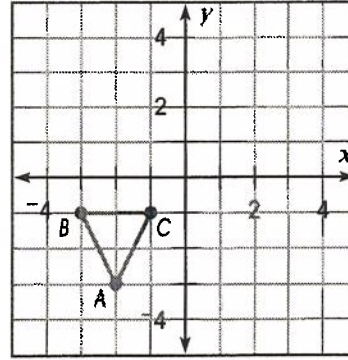
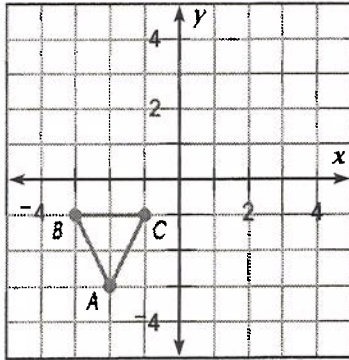
A rotation, or turn, is a type of transformation.

The preimage is rotated around a center of rotation. $\triangle ABC$ is rotated 90° counterclockwise about the origin. $\triangle A'B'C'$ is the image.



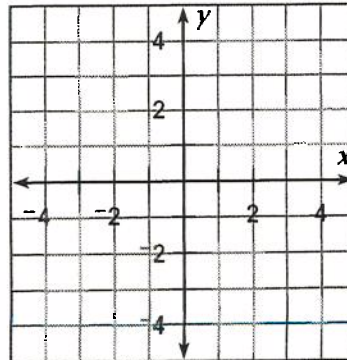
Draw the image for each rotation of $\triangle DEG$.

1. Rotate $\triangle ABC$ 180° counterclockwise about the origin.
2. Rotate $\triangle ABC$ 90° clockwise about the origin.



CRITICAL THINKING

Reflect the rotation image $\triangle A'B'C'$ from exercise 2 over the y -axis. Draw this new image. In which quadrant is the final image?



12 Dilations in the Coordinate Plane**Exercise 82***Lesson 12.9*

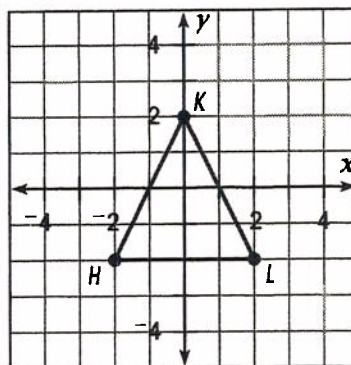
A dilation is also a type of transformation. In dilations, a similarity ratio is used to enlarge or reduce the preimage.

A similarity ratio of 2 means the dilation image is two times as large as the preimage.

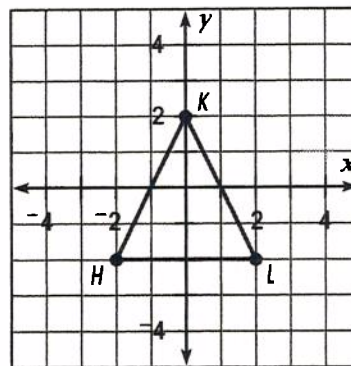
A similarity ratio of $\frac{1}{2}$ means the dilation image is one-half as large as the preimage.

Dilate $\triangle HKL$ with each similarity ratio. Draw $\triangle H'K'L'$.

1. similarity ratio of 2



2. similarity ratio of $\frac{1}{2}$

**CRITICAL THINKING**

Use grid paper. Dilate $\triangle HKL$ from above with a similarity ratio of 1. Draw $\triangle H'K'L'$. Explain what happens. Now, explain what happens when any preimage is dilated with a similarity ratio of 1.

12 Points and Distance in Space

Exercise 83

Lessons 12.10 and 12.11

A point in space has three coordinates, (x, y, z) .

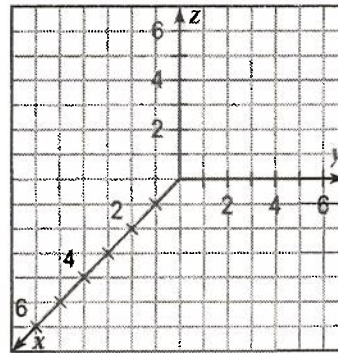
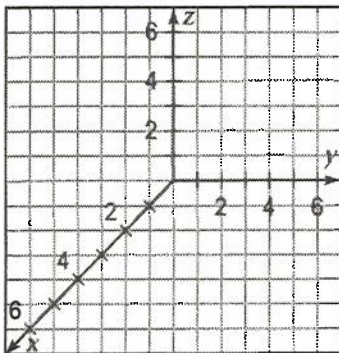
To find the distance between two points in space, use this formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Graph each point using the provided grid paper.

1. $J(3, 4, 5)$

2. $K(0, 3, 2)$



Find the distance between each pair of points in space.

3. $J(3, 4, 5)$ and $K(0, 3, 2)$

4. $L(1, -2, 0)$ and $M(3, 0, -4)$

CRITICAL THINKING

The distance between points A and B in space is $\sqrt{14}$. The coordinates of point A are $(-1, 4, z_1)$. The coordinates of point B are $(2, 3, z_2)$. Find the unknown coordinate z for both points A and B . (Hint: There are many possible answers.)

12 Midpoint of a Line Segment in Space**Exercise 84***Lesson 12.12*

To find the midpoint of a line segment in space, use this formula.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Find the coordinates of the midpoint of each line segment.

1. $A(2, 3, 1)$ and $B(6, 5, 3)$

2. $E(7, 0, 4)$ and $G(5, 4, 2)$

3. $H(0, 3, 4)$ and $J(2, 3, -2)$

4. $K(6, -6, -1)$ and $L(-4, 6, -1)$

5. $W(-4, -2, -2)$ and $F(5, 8, 0)$

6. $M(-7, 2, 4)$ and $P(3, -6, -4)$

CRITICAL THINKING

Point M is the midpoint of \overline{AB} in space. The coordinates of point M are $(8, 4, 6)$. The coordinates for point A are $(7, 5, 8)$. Find the coordinates for point B .

12 Problem-Solving Skill: Find the Resultant Vector

Exercise 85

Lesson 12.14

Solve each problem. Draw a diagram if you need to.

1. A ferry heads west across a river. It travels 20 miles per hour in still water. The river flows south at 15 miles per hour. In which direction will the ferry actually travel? How fast will it travel?
2. Juan flies a model airplane west at 8 miles per hour in still air. The wind is blowing north at 6 miles per hour. In which direction will the plane actually fly? How fast will it travel?
3. A hot-air balloon heads north. Its speed is 12 miles per hour in still air. The wind is blowing east at 5 miles per hour. In which direction will the balloon actually fly? How fast will it travel?
4. Juan and Maria are canoeing west across a river. They can paddle 12 miles per hour in still water. The river flows south at 9 miles per hour. In which direction will the canoe actually travel? How fast will it travel?

12

Problem-Solving Application: The Effect of Two Forces**Exercise 86***Lesson 12.15*

Solve each problem. Draw a diagram if you need to.

1. The speed of a ferry in still water is 25 miles per hour. It heads west across a river. The river flows south at $25\sqrt{3}$ miles per hour. In which direction will the ferry actually travel? How fast will it travel?

2. The speed of a model airplane in still air is 8 miles per hour. Jose wants to fly the plane west. The wind is blowing north at 8 miles per hour. In which direction will the model airplane actually fly? How fast will it travel?

3. The speed of a hot-air balloon in still air is 6 miles per hour. Jennifer wants to fly the balloon west. There is a wind blowing south at $6\sqrt{3}$ miles per hour. In which direction will the balloon actually fly? How fast will it travel?

4. The speed of a small plane in still air is 70 miles per hour. Malik flies the plane heading west. The wind blows north at 70 miles per hour. In which direction will the plane actually fly? How fast will it travel?

13 Trigonometric Ratios

Exercise 87

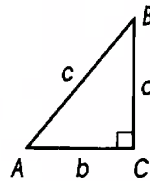
Lessons 13.1 and 13.2

Three Trigonometric Ratios

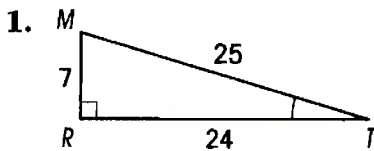
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



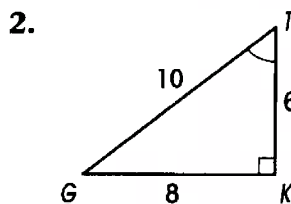
Write three trigonometric ratios for $\angle T$ for each triangle.



$$\sin T \text{ _____}$$

$$\cos T \text{ _____}$$

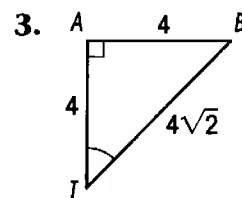
$$\tan T \text{ _____}$$



$$\sin T \text{ _____}$$

$$\cos T \text{ _____}$$

$$\tan T \text{ _____}$$



$$\sin T \text{ _____}$$

$$\cos T \text{ _____}$$

$$\tan T \text{ _____}$$

Write three trigonometric ratios for each angle. Use a Table of Trigonometric Ratios.

4. $\sin 70^\circ$ _____

$\cos 70^\circ$ _____

$\tan 70^\circ$ _____

5. $\sin 14^\circ$ _____

$\cos 14^\circ$ _____

$\tan 14^\circ$ _____

6. $\sin 57^\circ$ _____

$\cos 57^\circ$ _____

$\tan 57^\circ$ _____

Find the measure of each angle to the nearest degree. Use a Table of Trigonometric Ratios.

7. $\sin A = 0.7193$

8. $\cos B = 0.9205$

9. $\tan C = 2.4751$

CRITICAL THINKING

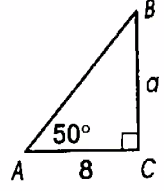
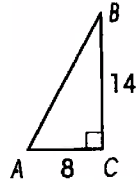
Draw three different 30° - 60° - 90° right triangles with the short leg 2 cm, 3 cm, and 4 cm. Write the sine ratio of the 30° angle in each triangle.

Will the $\sin 30^\circ$ always be $\frac{1}{2}$? Why or why not?

13 Tangent Ratio Exercise 88

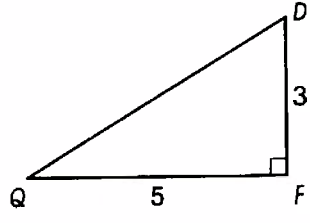
Lesson 13.3

<p>Find the measure of $\angle A$.</p> $\tan A = \frac{14}{8}$ $= 1.75$ $\tan 60^\circ = 1.7321$ <p>$\angle A$ is about 60°.</p>	<p>Find the length of side a.</p> $\tan 50^\circ = \frac{a}{8}$ $1.1918 = \frac{a}{8}$ $9.5 \approx a$ <p>The length of side a is about 9.5.</p>
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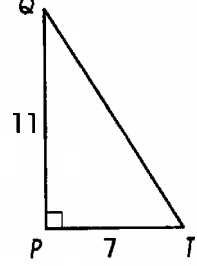
Find the measure of $\angle Q$ to the nearest degree. Use a Table of Trigonometric Ratios.

1.



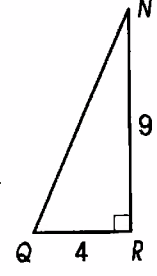
$\angle Q$ _____

2.



$\angle Q$ _____

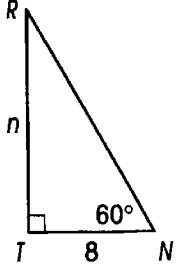
3.



$\angle Q$ _____

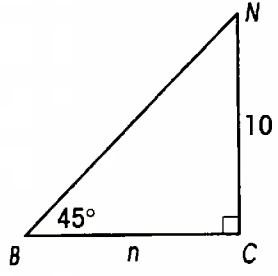
Find the value of n in each diagram. Use a Table of Trigonometric Ratios. Round the value to the nearest whole number.

4.



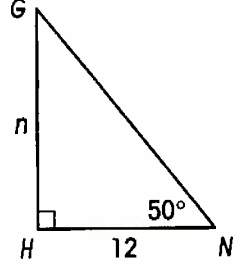
n _____

5.



n _____

6.



n _____

CRITICAL THINKING

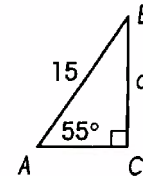
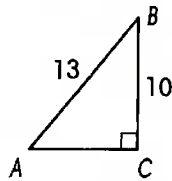
Can you use the tangent ratio to find the length of the hypotenuse? Why or why not?

13 Sine Ratio

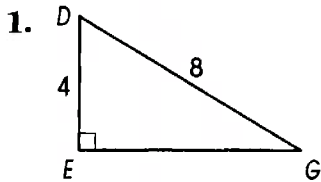
Exercise 89

Lesson 13.4

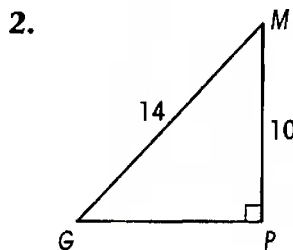
<p>Find the measure of $\angle A$.</p> $\sin A = \frac{10}{13}$ $= 0.7692$ $\sin 50^\circ = 0.7660$ <p>$\angle A$ is about 50°.</p>	<p>Find the length of side a.</p> $\sin 55^\circ = \frac{a}{15}$ $0.8192 = \frac{a}{15}$ $12 \approx a$ <p>The length of side a is about 12.</p>
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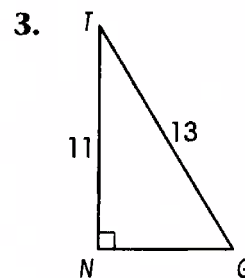
Find the measure of $\angle G$ to the nearest degree. Use a Table of Trigonometric Ratios.



$\angle G$ _____

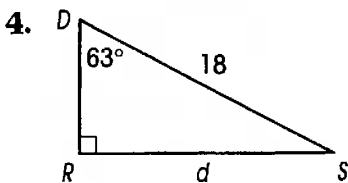


$\angle G$ _____

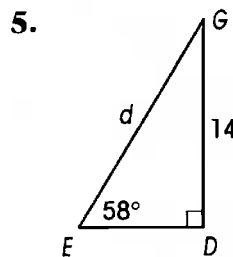


$\angle G$ _____

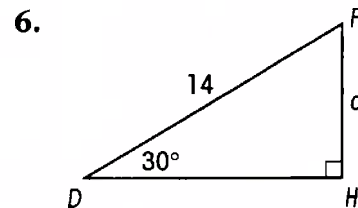
Find the value of d in each diagram. Use a Table of Trigonometric Ratios. Round the value to the nearest whole number.



d _____



d _____



d _____

CRITICAL THINKING

Explain why $\sin 30^\circ$ is 0.5000. Draw a diagram.

13 Cosine Ratio

Exercise 90

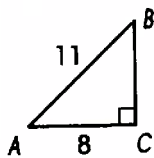
Lesson 13.5

Find the measure of $\angle A$.

$$\begin{aligned} \cos A &= \frac{8}{11} \\ &= 0.7273 \end{aligned}$$

$$\cos 44^\circ = 0.7193$$

$\angle A$ is about 44° .



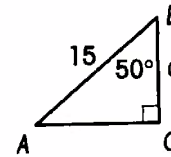
Find the length of side a .

$$\cos 50^\circ = \frac{a}{15}$$

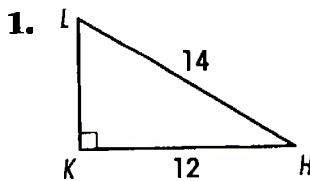
$$0.6428 = \frac{a}{15}$$

$$10 \approx a$$

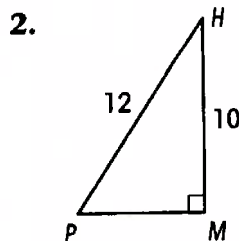
The length of side a is about 10.



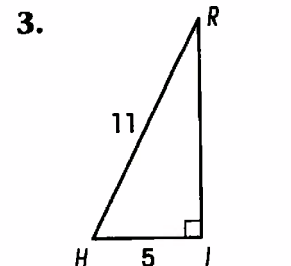
Find the measure of $\angle H$ to the nearest degree. Use a Table of Trigonometric Ratios.



$\angle H$ _____

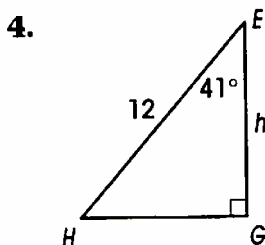


$\angle H$ _____

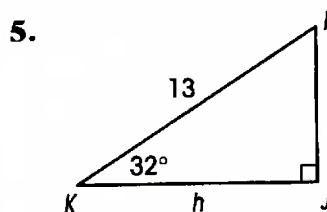


$\angle H$ _____

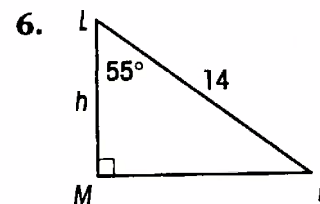
Find the value of h in each diagram. Use a Table of Trigonometric Ratios. Round the value to the nearest whole number.



h _____



h _____



h _____

CRITICAL THINKING

Use a Table of Trigonometric Ratios to find $\sin 30^\circ$ and $\cos 60^\circ$. Compare these two trigonometric ratios. What do you notice? Now, find $\sin 43^\circ$ and $\cos 47^\circ$. What do you notice? Choose another angle pair where the sine and cosine are the same. What kind of angles are these angle pairs?

13 Problem-Solving Skill: Angles of Elevation and Depression

Exercise 91

Lesson 13.7

Solve each problem. Round your answer to the nearest whole number. Draw a diagram if you need to.

1. Brian looks up at a house on top of a hill. The angle of elevation is 55° . The height of the hill is 350 feet. What is the distance between Brian and the house?
2. Esther and her friends fly a star balloon in a parade. The balloon is flown at a 75° angle of elevation. It uses a rope that is 20 feet long. The rope is held 3 feet above the ground. How high off the ground is the star balloon?
3. A plane is flying 5 miles above the ground. The horizontal distance from the airplane to the start of the runway is 19 miles. What is the angle of depression the airplane must use for its descent?
4. The height of a lighthouse is 50 feet. From the top of the lighthouse, the angle of depression of a boat is 40° . How far is the boat from the lighthouse?

13 Problem-Solving Application:
Using Trigonometric Ratios**Exercise 92***Lesson 13.8*

Solve each problem. Use trigonometric ratios. Round your answer to the nearest whole number.

1. A cable is used to support a radio tower. The cable makes a 65° angle with the ground and is 32 feet from the pole. How long is the cable?

2. A 15-foot ladder is leaning against a building. It makes a 74° angle with the ground. How far up the side of the building does the ladder reach?

3. A tree casts a shadow 40 feet long. The angle of elevation is 52° . What is the height of the tree?

4. A mountain is 4,450 feet tall. The lake at the mountain's base is 1,550 feet across. What is the angle of elevation from the farthest end of the lake to the top of the mountain?